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The **32**

MOST EFFECTIVE

SAT MATH

STRATEGIES

Second Edition

By Dr. Steve Warner

A Proven Roadmap To Your First-Choice College

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The 32 Most Effective SAT Math Strategies

A Proven Roadmap to
Your First-Choice College

Steve Warner, Ph.D.



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Second edition
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This book is dedicated to all my students over the past 12 years, I have learned just as much from all of you as you have learned from me.

I would also like to acknowledge Larry Ronaldson and Robert Folatico, thank you for introducing me to the rewarding field of SAT tutoring.

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SECRET 4

THE RIGHT WAY TO PREPARE: THE BEST 12 THINGS TO DO DURING THE TEST

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ote: *Every tip mentioned in this section should be followed every time you take a practice SAT as well as on the actual test. If you haven't trained yourself with these behaviors beforehand, it's extremely unlikely that you will use these behaviors when you take the actual exam.*

1. Leave at least 5 minutes towards the end of each section to check over easier problems.

It doesn't matter if you haven't finished the section. The last few problems on each math section are very difficult. If you have to leave some of these blank, then it's ok. It's better to spend your time looking for careless errors on problems that you can do than it is to waste your time on tricky problems that you are likely to get wrong. If you catch even one careless error per math section, your score can potentially go up *40 points*.

You should make sure that you are wearing a watch during the exam, of course.

2. Know the proper way to “check” your answers.

When you go back to check your earlier answers for careless errors *do not* simply look over your work to try to catch a mistake. This is usually a waste of time. Always redo the problem without looking at any of your previous work. Ideally, you want to use a different method than you

used the first time.

For example, if you solved the problem by picking numbers the first time, try to solve it algebraically the second time, or at the very least pick different numbers. If you don't know, or are not comfortable with a different method, then use the same method, but do the problem from the beginning and do not look at your original solution. If your two answers don't match up, then you know that this a problem you need to spend a little more time on to figure out where your error is.

This may seem time consuming, but that's ok. It's better to spend more time checking over a few problems, then to rush through a lot of problems and repeat the same mistakes.

3. The art of guessing.

Answering a multiple choice question wrong results in a $\frac{1}{4}$ point penalty. This is to discourage random guessing. If you have no idea how to do a problem, no intuition as to what the correct answer might be, and you can't even eliminate a single answer choice, then *DO NOT* just take a guess. Omit the question and move on.

If, however, you can eliminate even one answer choice, you should take a guess from the remaining four. You should of course eliminate as many choices as you can before you take your guess.

You are not penalized for getting a grid-in question wrong. Therefore you should always guess on grid-in questions that you don't know. Never leave any of these blank. If you have an idea of how large of a number the answer should be, then take a reasonable guess. If not, then just guess anything—don't think too hard—just put in a number.

4. How many questions should you attempt?

There are 3 math sections on the SAT. They can appear in any order. There is a 20 question multiple choice section, a 16 question multiple choice section, and an 18 question section that has 8 multiple choice questions and 10 grid-ins.

Let's call these sections A, B, and C, respectively. You should first make sure that you know what you got on your last SAT practice test, actual

SAT, or actual PSAT (whichever you took last). What follows is a general goal you should go for when taking the exam.

Score	Section A	Section B	Section C (Multiple choice)	Section C (Grid-in)
< 330	7/20	6/16	2/8	2/10
330 – 370	10/20	8/16	3/8	3/10
380 – 430	12/20	10/16	4/8	4/10
440 – 490	14/20	11/16	5/8	6/10
500 – 550	16/20	12/16	6/8	8/10
560 – 620	18/20	15/16	7/8	9/10
630 – 800	20/20	16/16	8/8	10/10

For example, a student with a current score of 450 should attempt the first 14 questions from section A, the first 11 questions from section B, the first 5 multiple choice questions from section C, and the first 6 grid-ins from section C.

This is *just* a general guideline. Of course it can be fine tuned. As a simple example, if you are particularly strong at number theory problems, but very weak at geometry problems, then you may want to try every number theory problem no matter where it appears, and you may want to reduce the number of geometry problems you attempt.

5. Relax—it's just a test.

If you don't do as well as you would like the first time, it's *not* the end of the world. You can take the test multiple times and many colleges will focus on your best scores. Also, the SAT is just one of many factors that a college uses in its decisions. So take the test seriously during the months before when you are preparing so that you can get the best score possible. But when test day comes, just go in there, relax, and do the best that you can.

6. Pace yourself.

Do not waste your time on a question that is too hard or will take too long. After you've been working on a question for about 1 minute you need to make a decision. If you understand the question and think that you can get the answer in another 30 seconds or so, continue to work on the problem. If you still don't know how to do the problem or you're using a technique that is going to take a long time, mark it off and come back to it later if you have time.

If you have eliminated at least one answer choice, or it is a grid-in, feel free to take a guess. But you still want to leave open the possibility of coming back to it later. Remember that every problem is worth the same amount. Don't sacrifice problems that you may be able to do by getting hung up on a problem that is too hard for you.

7. Mark questions that you need to come back to clearly.

If you are pacing yourself properly, then you will probably skip some questions that you may want to come back to if you have time. Make sure you mark these questions clearly so that you can get back to them quickly when it is appropriate to do so.

Also, be very careful when filling in answers that you're filling in the answer for the correct question number. Be especially careful when you're skipping questions.

8. Use but don't abuse your calculator.

You should definitely use your calculator for computations that would take longer by hand such as simple computations involving addition, subtraction, multiplication, division, exponentiation, and taking roots. You can also sometimes use the graphing capabilities of your calculator to solve certain problems, although very often this may take longer than other methods.

While we're on the subject of calculators, please make sure that your calculator has fresh batteries the day of the test. Nobody will supply one for you if yours dies. Also make sure that you are using a calculator that you are very comfortable with. Ideally you should have just one

calculator that you use whenever you are practicing SAT Math problems, and this should be the same calculator that you bring with you on test day. When a problem in this book has a solution using a calculator in its explanation it will be assumed that a TI-84 or comparable calculator is being used.

9. Never truncate or round calculator computations until the end of the problem.

If you are doing computations on your calculator always try to use the answer that your calculator has given to do later computations. Keep in mind that there is an ANS button on your calculator that uses the answer that your calculator produced last. Rounding or truncating too soon can lead to an incorrect final answer.

10. The proper way to write your answer on grid-in questions.

	/	/	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

The computer only grades what you have marked in the bubbles. The space above the bubbles is just for your convenience, and to help you do your bubbling correctly.

Never mark more than one circle in a column or the problem will automatically be marked wrong. You do not need to use all four columns. If you don't use a column just leave it blank.

11. What you can and can't answer on grid-ins.

The symbols that you can grid in are the digits 0 through 9, a decimal point, and a division symbol for fractions. Note that there is no negative symbol. So answers to grid-ins *cannot* be negative. Also, there are only four slots, so you can't get an answer such as 52,326.

Sometimes there is more than one correct answer to a grid-in question. Simply choose one of them to grid-in. *Never* try to fit more than one answer into the grid.

If your answer is a whole number such as 2451 or a decimal that only requires four or less slots such as 2.36, then simply enter the number starting at any column. The two examples just written must be started in the first column, but the number 16 can be entered starting in column 1, 2 or 3.

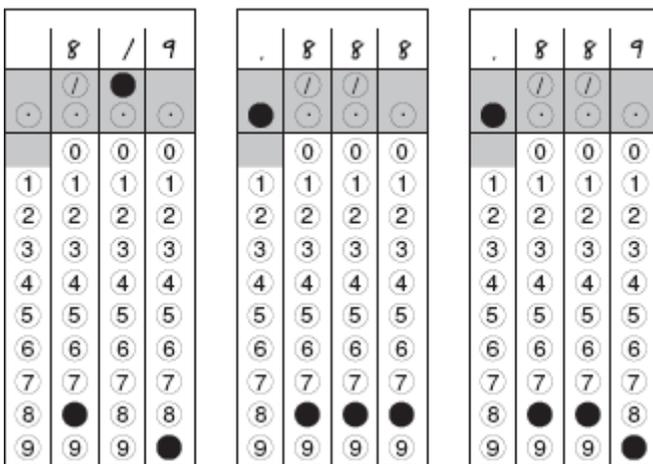
Note that there is no zero in column 1, so if your answer is 0 it must be gridded into column 2, 3 or 4.

Fractions can be gridded in any form as long as there are enough slots. The fraction 2/100 must be reduced to 1/50 simply because the first representation won't fit in the grid.

Fractions can also be converted to decimals before being gridded in. If a decimal cannot fit in the grid, then you can simply *truncate* it to fit. But you must use every slot in this case. For example, the decimal .16777777... can be gridded as .167, but .16 or .17 would both be marked wrong.

Instead of truncating decimals you can also *round* them. For example, the decimal above could be gridded as .168. Truncating is preferred because there is no thinking involved and you are less likely to make a careless error.

Here are three ways to grid in the number 8/9.



Never grid-in mixed numerals. If your answer is $2\frac{1}{4}$, and you grid in the mixed numeral $2\frac{1}{4}$, then this will be read as $21/4$ and will be marked wrong. You must either grid in the decimal 2.25 or the improper fraction $9/4$.

Here are two ways to grid in the mixed numeral $1\frac{1}{2}$ correctly.

	1	.	5
	/	/	
·	·	●	·
	0	0	0
1	●	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	●
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

	3	/	2
	/	●	
·	·	·	·
	0	0	0
1	1	1	1
2	2	2	●
3	●	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

12. Time is not as much of an issue as you think.

Remember that you don't need to attempt all of the problems unless your current ability level is a 630 or higher. If you stick to the number of suggested problems above, then you will have plenty of time during the test to do the allotted number of problems carefully and to check many of them over.

Don't stress out over time. This is counterproductive and the fact is that if you're taking the test right there is plenty of time.

STRATEGY 20

CHANGE AVERAGES TO SUMS

A

problem involving averages often becomes much easier when we first convert the averages to sums. We can easily change an average to a sum using the following simple formula.

$$\text{Sum} = \text{Average} * \text{Number}$$

Many problems with averages involve one or more conversions to sums, followed by a subtraction.

Note: The above formula comes from eliminating the denominator in the definition of average.

$$\text{Average} = \frac{\text{Sum}}{\text{Number}}$$

LEVEL 1: PROBABILITY & STATISTICS

77. The average (arithmetic mean) of three numbers is 100. If two of the numbers are 80 and 130, what is the third number?

- (A) 70
- (B) 80
- (C) 90
- (D) 100
- (E) 110

* In this case we are averaging 3 numbers. Thus the Number is 3. The Average is given to be 100. So the Sum of the 3 numbers is $100 \times 3 = 300$. Since we know that two of the numbers are 80 and 130, the third number is $300 - 80 - 130 = 90$, choice (C).

LEVEL 3: PROBABILITY & STATISTICS

78. The average (arithmetic mean) of seven numbers is 10. When an eighth number is added, the average of the eight numbers is also 10. What is the eighth number?

- (A) 0
- (B) $\frac{4}{5}$
- (C) $\frac{5}{4}$
- (D) 8
- (E) 10

* The sum of the seven numbers is $10 \times 7 = 70$.
The sum of the eight numbers is $10 \times 8 = 80$.
The eighth number is $80 - 70 = 10$, choice (E).

LEVEL 4: PROBABILITY & STATISTICS

79. If the average (arithmetic mean) of k and $k+3$ is b and if the average of k and $k-3$ is c , what is the average of b and c ?

- (A) 1
- (B) $\frac{k}{2}$
- (C) k
- (D) $k + \frac{1}{2}$
- (E) $2k$

* The sum of k and $k+3$ is $k + (k+3) = 2k+3$, so that $2k+3 = 2b$. The sum of k and $k-3$ is $k + (k-3) = 2k-3$ so that $2k-3 = 2c$. So,

$$\begin{aligned} 2b + 2c &= 4k \\ 2(b + c) &= 4k \\ (b + c)/2 &= k. \end{aligned}$$

Thus, the answer is choice (C).

Alternative solution by picking numbers: Let's choose a value for k , say $k = 5$. Then $k+3 = 8$ and $k-3 = 2$. So,

$$\begin{aligned} b &= (5+8)/2 = 13/2 = 6.5 \\ c &= (5+2)/2 = 7/2 = 3.5 \end{aligned}$$

and the average of b and c is $(b+c)/2 = (6.5+3.5)/2 = 10/2 = 5$. **Put a nice big, dark circle around this number so that you can find it easily later.** We now substitute $k = 5$ into each answer choice.

- (A) 1
- (B) 2.5
- (C) 5
- (D) 5.5
- (E) 10

We now compare each of these numbers to the number that we put a nice big, dark circle around. Since (A), (B), (D) and (E) are incorrect we can eliminate them. Therefore the answer is choice (C).

Important note: (C) is **not** the correct answer simply because it is equal to 5. It is correct because all 4 of the other choices are **not** 5. **You absolutely must check all five choices!**

LEVEL 5: PROBABILITY & STATISTICS

80. The average (arithmetic mean) salary of employees at a bank with A employees in thousands of dollars is 53, and the average salary of employees at a bank with B employees in thousands of dollars is 95. When the salaries of both banks are combined, the average salary in thousands of dollars is 83.

What is the value of $\frac{A}{B}$?

* The sum of the salaries of employees at bank A (in thousands) is $53A$.
The sum of the salaries of employees at bank B (in thousands) is $95B$.

Adding these we get the sum of the salaries of all employees (in thousands): $53A + 95B$.

We can also get this sum directly from the problem.

$$83(A + B) = 83A + 83B.$$

So we have that $53A + 95B = 83A + 83B$.

We get A to one side of the equation by subtracting $53A$ from both sides, and we get B to the other side by subtracting $83B$ from both sides.

$$12B = 30A$$

We can get A/B to one side by performing **cross division**. We do this just like cross multiplication, but we divide instead. Dividing both sides of the

equation by 30B will do the trick (this way we get rid of B on the left, and 30 on the right).

$$A/B = 12/30 = 2/5$$

So we can grid in **2/5** or **.4**.

LEVEL 5: PROBABILITY AND STATISTICS

81. A group of students take a test and the average score is 65. One more student takes the test and receives a score of 92 increasing the average score of the group to 68. How many students were in the initial group?

* Let n be the number of students in the initial group. We change the average to a sum using the formula $\text{Sum} = \text{Average} * \text{Number}$

So the initial sum is $65n$.

When we take into account the new student, we can find the new sum in two different ways.

(1) We can add the new score to the old sum to get $65n + 92$.

(2) We can compute the new sum directly using the simple formula above to get $68(n + 1) = 68n + 68$.

We now set these equal to each other and solve for n :

$$65n + 92 = 68n + 68$$

$$24 = 3n$$

$$n = 8.$$

AFTERWORD

YOUR ROAD TO SUCCESS



Congratulations! You are now much better prepared in SAT Math—and this preparation is going to show when you take your next practice tests and on the actual exam. You should be extremely proud of this accomplishment. You should feel much more confident walking into the exam.

Learning the strategies and doing the problems in this book will have boosted your SAT score and ultimately this means you will have a greater selection when choosing colleges.

Just by doing the problems in this book, understanding your errors, and redoing the problems you get wrong over and over until you can get them right on your own, you will be increasing your level of mathematical maturity, and increasing your SAT math score.

If you want to improve your score even more, you should reread the book focusing on the methods that you couldn't understand the first time around. Keep redoing those problems you can't quite get yourself until you can do them on your own. Once you can do the problems using the basic strategies presented here, try to solve them again using the more advanced methods presented. This will increase your mathematical maturity even further, and thus give an even greater improvement in your score.

After completing this book you should continue to do SAT Math problems for 10 to 20 minutes each day right up until the day of the SAT (but take off the day before the test). Try to stick mostly with problems created by the College Board and the author of this book. These will most closely reflect questions that will appear on the actual exam. Try to

do problems that are at and slightly above your current ability level in each of the four major subject areas.

Try to complete at least four practice tests before taking the actual exam. For at least two of them you should take the whole test in one sitting (all Math, Critical Reading and Writing sections). Remember to follow the advice given in the introduction when preparing for the exam (Secret 3) and when actually taking the exam (Secret 4).

I really want to thank you for putting your trust in me. I realize how big a deal this is, and I want to assure you that you have made excellent use of your time by studying with this book. I wrote every sentence thinking how I can I increase your score with the minimum amount of effort. This book represents my ten plus years of experience as an SAT math tutor.

Nothing gives me greater pleasure then when I receive e-mails and phone calls from former students telling me how well they did on their SAT, or telling me that they made it into the college of their choice.

Writing this book has given me that same feeling of pleasure because I know that many more students are going to benefit from my experience even though I will never get to meet them all face to face.

Remember that you control your own destiny. Continue to work hard, and work smart. By using a little creativity you can usually accomplish any task you are given with a minimum amount of effort. Have confidence in yourself in everything you do and you will succeed. I wish you the best of luck on the SAT, on getting into your choice college, and in life.

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About the Author

Steve Warner, a New York native, earned his Ph.D. at Rutgers University in Pure Mathematics in May, 2001. While a graduate student, Dr. Warner won the TA Teaching Excellence Award.



After Rutgers, Dr. Warner joined the Penn State Mathematics Department as an Assistant Professor. In September, 2002, Dr. Warner returned to New York to accept an Assistant Professor position at Hofstra University. By September 2007, Dr. Warner had received tenure and was promoted to Associate Professor. He has taught undergraduate and graduate courses in Precalculus, Calculus, Linear Algebra, Differential Equations,

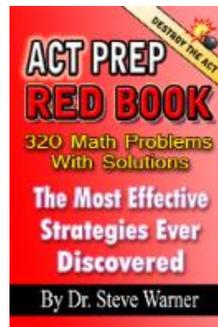
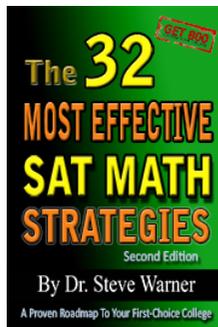
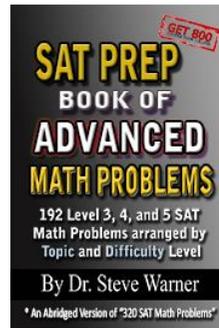
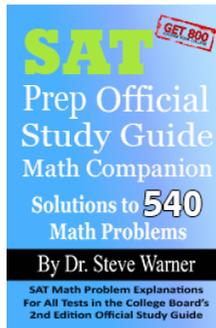
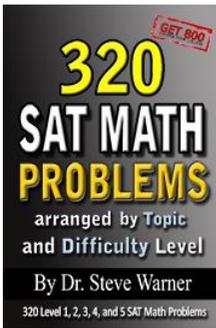
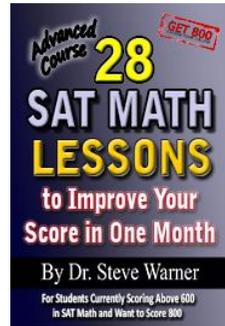
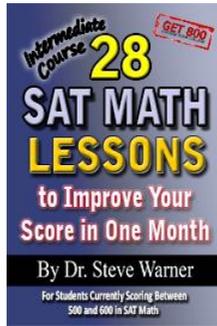
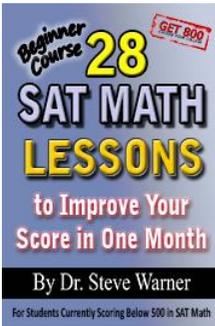
Mathematical Logic, Set Theory and Abstract Algebra.

Over that time, Dr. Warner participated in a five year NSF grant, “The MSTP Project,” to study and improve mathematics and science curriculum in poorly performing junior high schools. He also published several articles in scholarly journals, specifically on Mathematical Logic.

Dr. Warner has over 15 years of experience in general math tutoring and over 10 years of experience in SAT math tutoring. He has tutored students both individually and in group settings.

Currently Dr. Warner lives in Staten Island with his two cats, Achilles and Odin. Since the age of 4, Dr. Warner has enjoyed playing the piano—especially compositions of Chopin as well as writing his own music. He also maintains his physical fitness through weightlifting.

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CHOOSE YOUR COLLEGE

Put your first-choice college well within your reach with *The 32 Most Effective SAT Math Strategies* - a powerful collection of the most clever and easy-to-follow problem solving methods and tips that will significantly boost your score in SAT Math. After 10 rewarding years of SAT math tutoring, these strategies have been proven to work for any student, even if you're convinced that math is not your strong subject. When studying for the SAT it is always better to work smart than to work hard. This book is specifically designed to give the maximum result with the minimum amount of effort.

Using the proven tips, tactics, and techniques in this book you'll spend less time on each problem, you'll answer more difficult problems correctly, and you'll feel confident knowing you're applying a trusted system to one of the most important tests you will ever take.

Inside this book you will find basic advice that will significantly increase your SAT score before you even pick up your pencil. The main part of the book consists of 32 simple but powerful strategies together with over 100 problems complete with multiple solutions.

Here's to your success on the SAT, in college, and in life.



Steve Warner earned his Ph.D. at Rutgers University in May, 2001, and he currently works as an Associate Professor of Mathematics at Hofstra University. He has over 15 years of experience in general math tutoring and over 10 years of experience in SAT math tutoring