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LESSON 1
NUMBER THEORY

Start with the Middle Answer Choice

In many ACT math problems, you can get the answer simply by trying each of the answer choices until you find the one that works. Unless you have some intuition as to what the correct answer might be, then you should always start with the middle answer choice (C or H) as your first guess (an exception will be detailed in the next strategy below). The reason for this is simple. Answers are very often (but not always) given in increasing or decreasing order. So, if the middle choice fails you can sometimes eliminate two of the other choices as well.

Try to answer the following question using this strategy. **Do not** check the solution until you have attempted this question yourself.

LEVEL 1: NUMBER THEORY

1. Three consecutive integers are listed in increasing order. If their sum is 138, what is the second integer in the list?

   A. 45
   B. 46
   C. 47
   D. 48
   E. 49

Solution

Begin by looking at choice C. If the second integer is 47, then the first integer is 46 and the third integer is 48. Therefore, we get a sum of $46 + 47 + 48 = 141$. This is a little too big. So, we can eliminate choices C, D, and E.

We next try choice B. If the second integer is 46, then the first integer is 45 and the third integer is 47. So, the sum is $45 + 46 + 47 = 138$. Thus, the answer is choice **B**.

Remark: You should use your calculator to compute these sums. This will be quicker and you are less likely to make a careless error.

Before we go on, try to solve this problem in two other ways.

(1) Algebraically (the way you would do it in school).
(2) With a single computation.

Here is a hint for method (2):

Hint: In a set of consecutive integers, the average (arithmetic mean) and median are equal (see Lesson 4 for definitions of average and median).

Important Note: If you have trouble understanding the following solutions, it is okay. Just do your best to follow the given explanations.

Solutions

(1) An algebraic solution: If we name the least integer x, then the second and third integers are $x + 1$ and $x + 2$, respectively. So, we have

$$x + (x + 1) + (x + 2) = 138$$
$$3x + 3 = 138$$
$$3x = 135$$
$$x = 45$$

The second integer is $x + 1 = 46$, choice **B**.

Important: Always remember to check what the question is asking for before choosing your answer. Many students would accidently choose choice A here as soon as they discovered that $x = 45$.

Note: The following is a bit more efficient.

$$x + (x + 1) + (x + 2) = 138$$
$$3x + 3 = 138$$
$$3(x + 1) = 138$$
$$x + 1 = 46$$

*** (2) A quick, clever solution:** Simply divide 138 by 3 to get 46, choice **B**.

When NOT to Start with the Middle Answer Choice

If the word **least** appears in the problem, then start with the smallest number as your first guess. Similarly, if the word **greatest** appears in the problem, then start with the largest number as your first guess.

Try to answer the following question using this strategy. **Do not** check the solution until you have attempted this question yourself.

LEVEL 2: NUMBER THEORY

2. What is the least positive integer divisible by the integers 3, 7 and 14?

   F. 168
   G. 126
   H. 84
   J. 42
   K. 28

Solution

Begin by looking at choice K since it is the smallest. $28/3$ comes to approximately 9.33 in our calculator. Since this is not an integer, 28 is **not** divisible by 3. We can therefore eliminate choice K. We next try choice J.

$$42/3 = 14 \quad 42/7 = 6 \quad 42/14 = 3$$

Since these are all integers, the answer is choice **J**.

Before we go on, try to solve this problem directly (without using the answer choices).

Solutions

(1) The question is asking for the **least common multiple** of 3, 7 and 14. Here is one way to find it.

Step 1: Find the prime factorization of each integer in the set.

$$3 = 3$$
$$7 = 7$$
$$14 = 2 \cdot 7$$

Step 2: Choose the highest power of each prime that appears in any of the factorizations.

2, 3 and 7 (in this example the highest power of each prime is 1)

Step 3: Multiply these numbers together to get the least common multiple.

$2 \cdot 3 \cdot 7 = 42$, choice **J**.

*** (2) Getting the answer quickly:** Starting from 3, write down the prime factors of each number, skipping any that do not contribute to the least common multiple. So, we would write 3, then 7. We would then think of 14 as $2 \cdot 7$, and so we would write down 2 (we do not write 7 again because we have already written it). So, we have 3 7 2. We then multiply these numbers together to get $3 \cdot 7 \cdot 2 = 42$, choice **J**.

(3) Calculator solution: We use the **lcm** feature on our graphing calculator (found under NUM after pressing the MATH button). Our calculator can only handle two numbers at a time. So, compute $\mathbf{lcm}(3, 7) = 21$, and then $\mathbf{lcm}(21, 14) = 42$ choice **J.**

You're doing great! Let's just practice a bit more. Try to solve each of the following problems by using one of the two strategies you just learned. Then, if possible, solve each problem another way. The answers to these problems, followed by full solutions are at the end of this lesson. **Do not** look at the answers until you have attempted these problems yourself. Please remember to mark off any problems you get wrong.

LEVEL 1: NUMBER THEORY

3. Which of the following numbers is less than 0.416?

   A. 0.4106
   B. 0.4161
   C. 0.4166
   D. 0.42
   E. 0.421

4. Which of the following numbers disproves the statement "A number that is divisible by 4 and 8 is also divisible by 12"?

   F. 24
   G. 48
   H. 56
   J. 72
   K. 96

5. The absolute value of which of the following numbers is the greatest?

A. -0.7
B. -0.073
C. -0.0079
D. 0.07
E. 0.078

6. Which of the following numbers is NOT a factor of 252 ?

F. 6
G. 14
H. 27
J. 42
K. 63

7. What is the least integer greater than $\sqrt{67}$?

A. 7
B. 8
C. 9
D. 10
E. 11

LEVEL 2: NUMBER THEORY

8. What is the greatest positive integer that is a divisor of 14, 49, and 63?

F. 1
G. 3
H. 5
J. 7
K. 14

9. Among the following rational numbers, which has the greatest value?

A. 0.25
B. $0.2\overline{5}$
C. $0.\overline{25}$
D. 0.252
E. 0.2507

LEVEL 3: NUMBER THEORY

10. What is the largest positive integer value of k for which 3^k divides 18^4?

F. 2
G. 4
H. 6
J. 7
K. 8

Definitions Used in This Lesson

The **integers** are the counting numbers together with their negatives.

$$\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$$

The **positive integers** consist of the positive numbers from that set.

$$\{1, 2, 3, 4, \dots\}$$

Consecutive integers are integers that follow each other in order. The difference between consecutive integers is 1. Here are two examples.

$1, 2, 3$	these are three consecutive integers
$-3, -2, -1, 0, 1$	these are five consecutive integers

In general, if x is an integer, then x, $x + 1$, $x + 2$, $x + 3$, … are consecutive integers.

An integer n is **divisible** by an integer d if there is another integer k such that $n = dk$. For example, 42 is divisible by 7 because $42 = 7 \cdot 6$. In practice, we can check if n is divisible by d simply by dividing n by d in our calculator. If the answer is an integer, then n is divisible by d. If the answer is not an integer (it contains digits after the decimal point), then n is not divisible by d. If n is divisible by d, we say that d is a **divisor** (or **factor**) of n. We also say that n is a **multiple** of d.

The **least common multiple (lcm)** of a set of positive integers is the smallest positive integer that is divisible by each integer in the set.

The **greatest common divisor (gcd)** of a set of positive integers is the largest positive integer that each integer in the set is divisible by.

Answers

1. B	6. H
2. J	7. C
3. A	8. J
4. H	9. B
5. A	10. K

Full Solutions

3.

We can compare two decimals by looking at the first position where they disagree. For example, 0.415 is less than 0.416 because 5 is less than 6. If a digit is missing, there is a hidden 0 there. Thus, 0.4 is also less than 0.416 because 0.4 is the same as 0.400 and 0 is less than 1 (remember that we look at the **first** position where the decimals disagree). Since 0 is less than 6, the answer is choice **A**.

*** Quick Solution:** Answers are often given in increasing or decreasing order on the ACT (in this problem they are given in increasing order). Therefore, choice **A** is the only reasonable answer.

Remark: The words "less than" would seem to indicate we should start by looking at the smallest answer choice. In this case that is choice A. Note that we have essentially used the second strategy from this lesson here (we started with the smallest answer choice).

4.

*** Solution by starting with choice H:** We want a number that is divisible by 4 and 8, but **not** by 12. Use your calculator and begin with choice H. When we divide 56 by 4, 8 and 12 we get 14, 7 and approximately 4.67. Since 14 and 7 are integers we see that 56 is divisible by 4 and 8. Since 4.67 is **not** an integer, 56 is not divisible by 12. Therefore, the answer is choice **H**.

5.

***** A quick glance at the answer choices shows that they are listed in increasing order. Since we are looking at the absolute value of the choices, the answer must be either choice A or E. The absolute value of -0.7 is 0.7, and the absolute value of 0.078 is 0.078. Since 0.7 is greater than 0.078, the answer is choice **A**.

Notes: (1) When we take the absolute value of a real number, we simply remove the minus sign if there is one (otherwise we do nothing). So, the absolute values of the choices, beginning with choice A, are 0.7, 0.073, 0.0079, 0.07, and 0.078.

(2) Although it is not necessary to understand this problem, the standard notation for taking the absolute value of a number is to put the number between two vertical lines. So, for example, the absolute value of -0.7 can be written as $|-0.7|$, and we have $|-0.7| = 0.7$.

6.

*** Solution by starting with choice H:** We divide 252 by 27 in our calculator to get approximately 9.333. Since this is not an integer, 27 is not a factor of 252, and the answer is choice **H**.

Notes: (1) The prime factorization of 252 is $2^2 \cdot 3^2 \cdot 7$. So, a factor of 252 has at most two factors of 2 and 3, and at most one factor of 7 (and no other prime factors).

(2) The factors of 252 are 1, 2, $2^2 = 4$, 3, $2 \cdot 3 = 6$, $2^2 \cdot 3 = 12$, $3^2 = 9$, $2 \cdot 3^2 = 18$, $2^2 \cdot 3^2 = 36$, 7, $2 \cdot 7 = 14$, $2^2 \cdot 7 = 28$, $3 \cdot 7 = 21$, $2 \cdot 3 \cdot 7 = 42$, $2^2 \cdot 3 \cdot 7 = 84$, $3^2 \cdot 7 = 63$, $2 \cdot 3^2 \cdot 7 = 126$, and $2^2 \cdot 3^2 \cdot 7 = 252$.

7.

Solution by starting with choice A: Since the word "least" appears in the problem, let's start with the smallest answer choice, choice A. We have $7^2 = 49$. This is too small. Let's try choice B. We have $8^2 = 64$, still a bit too small. So, the answer is most likely choice C.

Let's check: $9^2 = 81$. Since $8^2 < 67$, and $9^2 > 67$, the answer is **C.**

*** Quick solution:** If we take the square root of 67 in our calculator we get approximately 8.185. The least integer greater than this is 9, which is choice **C.**

8.

*** Solution by starting with choice K:** Pull out your calculator. Since the question has the word **"greatest"** in it, we will start with the greatest answer choice, which is choice K, and we will divide each of the three numbers by 14. Since 49 divided by 14 is 3.5 (not an integer), choice K is not the answer. We next try choice J. The divisions give us 2, 7 and 9, respectively. Since these are all integers, the answer is choice **J**.

Note that the three given integers are all divisible by 1, but choice A is not the answer because 7 is greater.

Direct solution: We are being asked to find the **greatest common divisor** of 14, 49 and 63, which is 7, choice **J**.

Finding the greatest common divisor:

Here are two ways to find the greatest common divisor of the given integers.

(1) List all divisors of each integer and look for the biggest one they have in common.

Divisors of 14: $\{1, 2, 7, 14\}$
Divisors of 49: $\{1, 7, 49\}$
Divisors of 63: $\{1, 3, 7, 9, 21, 63\}$
Common Divisors: $\{1, 7\}$

Thus, the greatest common divisor is 7.

(2) Here is a more sophisticated method (this method is much quicker if the given integers are large).

Step 1: Find the prime factorization of each number in the set.

$$14 = 2 \cdot 7$$
$$49 = 7^2$$
$$63 = 3^2 \cdot 7$$

Step 2: Choose the lowest power of each prime that appears in **all** the factorizations. In this case, this is just 7.

Step 3: Multiply these numbers together to get the greatest common divisor. (In this case there is nothing to multiply since there is only one prime factor that the three integers have in common.)

Remark: We can also write the above prime factorizations as follows:

$$14 = 2^1\ 3^0\ 7^1$$
$$49 = 2^0\ 3^0\ 7^2$$
$$63 = 2^0\ 3^2\ 7^1$$

It is easy to see in this form that the lowest power of 2 is $2^0 = 1$, and similarly the lowest power of 3 is $3^0 = 1$.

9.

* $0.2\overline{5} = 0.255555\ldots$ and $0.\overline{25} = 0.252525\ldots$ We see that the greatest value is choice **B**.

10.

*** Solution by starting with choice K:** Pull out your calculator. Since the question has the word **"largest"** in it, we will start with the largest answer choice which is choice K, and we will divide 18^4 by 3^8. We type $18\text{^}4 \,/\, 3\text{^}8$ into our calculator and the output is 16. Since 16 is an integer, the answer is choice **K**.

Note that all five answer choices give an integer, but 8 is the largest positive integer that works.

Direct solution: The prime factorization of 18 is $18 = 2 \cdot 3^2$. Therefore,

$$18^4 = (2 \cdot 3^2)^4 = 2^4(3^2)^4 = 2^4 \cdot 3^8.$$

From this prime factorization, it should be clear that 3^8 divides 18^4, but 3^9 does not, choice **K**.

For a review of the basic laws of exponents used here see Lesson 10.

OPTIONAL MATERIAL

The following questions will test your understanding of definitions used in this lesson. These are **not** in the format of ACT questions.

1. Which of the following numbers are integers? Choose all that apply.

   $\frac{1}{2}$ $\quad -3 \quad .67 \quad \sqrt{2} \quad 0 \quad 1800 \quad 1.1 \quad \sqrt{4} \quad \frac{18}{3} \quad \frac{\sqrt{18}}{\sqrt{2}} \quad \pi$

2. List 10 consecutive integers beginning with -6. Which of these are positive integers?

Answers

1. $-3,\ 0,\ 1800,\ \sqrt{4} = 2,\ \frac{18}{3} = 6, \frac{\sqrt{18}}{\sqrt{2}} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3$
2. $-6, -5, -4, -3, -2, -1, 0, 1, 2, 3$ positive integers: $1, 2, 3$

About the Author

Dr. Steve Warner, a New York native, earned his Ph.D. at Rutgers University in Pure Mathematics in May, 2001. While a graduate student, Dr. Warner won the TA Teaching Excellence Award.

After Rutgers, Dr. Warner joined the Penn State Mathematics Department as an Assistant Professor. In September, 2002, Dr. Warner returned to New York to accept an Assistant Professor position at Hofstra University. By September 2007, Dr. Warner had received tenure and was promoted to Associate Professor. He has taught undergraduate and graduate courses in Precalculus, Calculus, Linear Algebra, Differential Equations, Mathematical Logic, Set Theory and Abstract Algebra.

Over that time, Dr. Warner participated in a five year NSF grant, "The MSTP Project," to study and improve mathematics and science curriculum in poorly performing junior high schools. He also published several articles in scholarly journals, specifically on Mathematical Logic.

Dr. Warner has more than 15 years of experience in general math tutoring and tutoring for standardized tests such as the SAT, ACT and AP Calculus exams. He has tutored students both individually and in group settings.

In February, 2010 Dr. Warner released his first SAT prep book "The 32 Most Effective SAT Math Strategies," and in 2012 founded Get 800 Test Prep. Since then Dr. Warner has written books for the SAT, ACT, SAT Math Subject Tests and AP Calculus exams.

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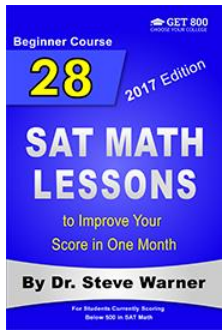

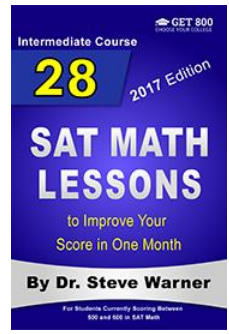

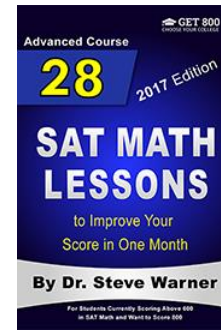

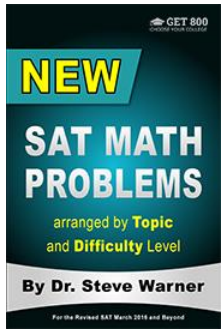

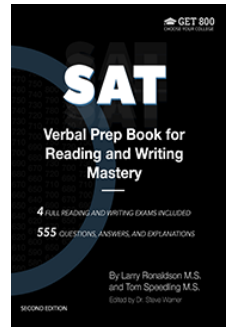

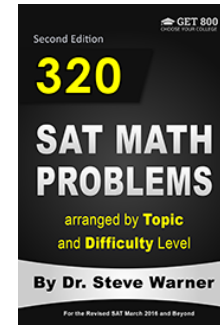

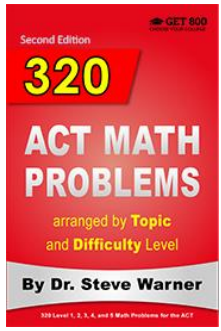

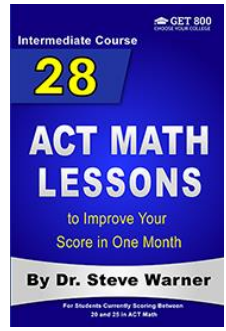

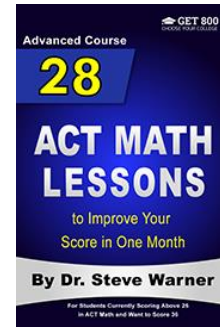

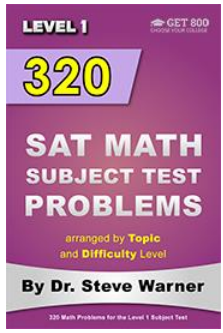
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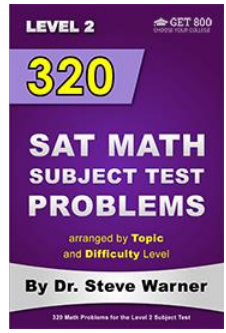
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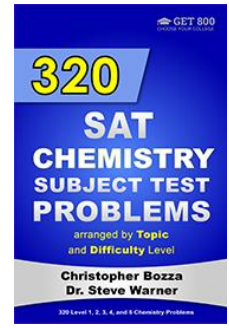
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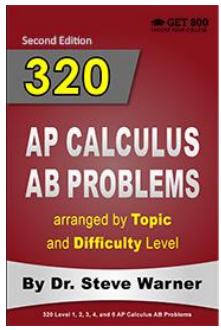
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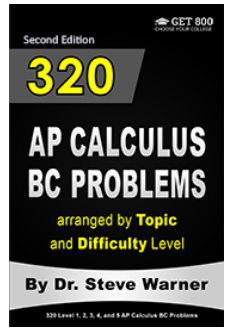
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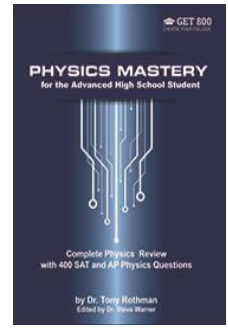
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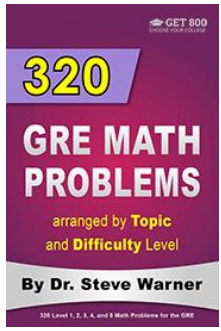
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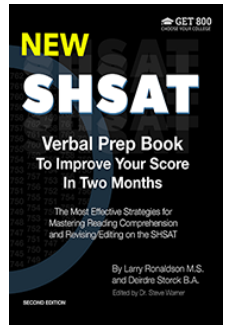
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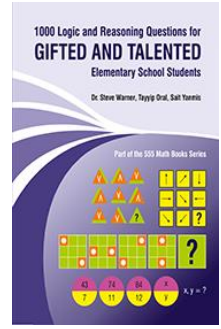
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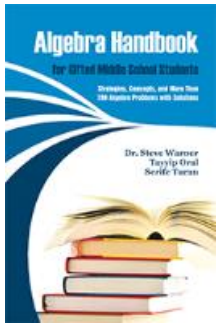

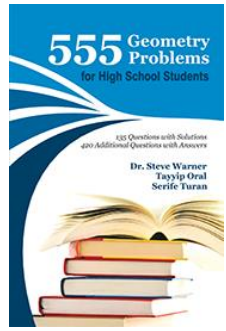

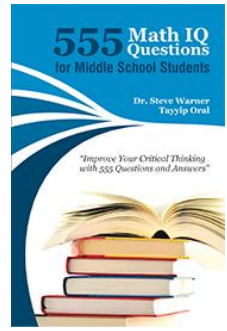

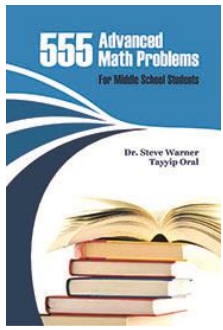

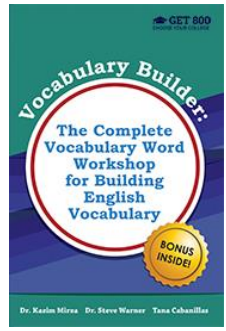

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