

Second Edition

**320**

# ACT MATH PROBLEMS

arranged by **Topic**  
and **Difficulty** Level

**By Dr. Steve Warner**

320 Level 1, 2, 3, 4, and 5 Math Problems for the ACT

## Legal Notice

This book is copyright 2016 with all rights reserved. It is illegal to copy, distribute, or create derivative works from this book in whole or in part or to contribute to the copying, distribution, or creating of derivative works of this book.

**FREE  
SAMPLE**

# **320 ACT Math Problems arranged by Topic and Difficulty Level**

320 Level 1, 2, 3, 4, and 5 Math  
Problems for the ACT

---

Dr. Steve Warner



© 2015, All Rights Reserved  
Second edition  
ACTPrepGet36.com © 2013

## **BOOKS BY DR. STEVE WARNER FOR COLLEGE BOUND STUDENTS**

28 New SAT Math Lessons to Improve Your Score in One Month

Beginner Course

Intermediate Course

Advanced Course

New SAT Math Problems arranged by Topic and Difficulty Level

New SAT Verbal Prep Book for Reading and Writing Mastery

320 SAT Math Subject Test Problems

Level 1 Test

Level 2 Test

The 32 Most Effective SAT Math Strategies

SAT Prep Official Study Guide Math Companion

SAT Vocabulary Book

320 ACT Math Problems arranged by Topic and Difficulty Level

320 SAT Math Problems arranged by Topic and Difficulty Level

320 AP Calculus AB Problems

320 AP Calculus BC Problems

555 Math IQ Questions for Middle School Students

555 Advanced Math Problems for Middle School Students

555 Geometry Problems for High School Students

Algebra Handbook for Gifted Middle School Students

## **CONNECT WITH DR. STEVE WARNER**



## Table of Contents

<b>Introduction: The Proper Way to Prepare</b>	<b>7</b>
<b>1. Using this book effectively</b>	<b>7</b>
<b>2. The magical mixture for success</b>	<b>8</b>
<b>3. Practice problems of the appropriate level</b>	<b>9</b>
<b>4. Practice in small amounts over a long period of time</b>	<b>10</b>
<b>5. Redo the problems you get wrong over and over and over until you get them right</b>	<b>10</b>
<b>6. Check your answers properly</b>	<b>11</b>
<b>7. Take a guess whenever you cannot solve a problem</b>	<b>11</b>
<b>8. Pace yourself</b>	<b>11</b>
<b>9. Attempt the right number of questions</b>	<b>12</b>
<b>10. Use your calculator wisely</b>	<b>13</b>
<b>Problems by Level and Topic with Fully Explained Solutions</b>	<b>17</b>
<b>Level 1: Number Theory</b>	<b>15</b>
<b>Level 1: Algebra and Functions</b>	<b>20</b>
<b>Level 1: Geometry</b>	<b>26</b>
<b>Level 1: Probability and Statistics</b>	<b>32</b>
<b>Level 2: Number Theory</b>	<b>37</b>
<b>Level 2: Algebra and Functions</b>	<b>43</b>
<b>Level 2: Geometry</b>	<b>49</b>
<b>Level 2: Probability and Statistics</b>	<b>55</b>
<b>Level 2: Trigonometry</b>	<b>58</b>
<b>Level 3: Number Theory</b>	<b>63</b>
<b>Level 3: Algebra and Functions</b>	<b>68</b>
<b>Level 3: Geometry</b>	<b>74</b>
<b>Level 3: Probability and Statistics</b>	<b>83</b>
<b>Level 3: Trigonometry</b>	<b>85</b>
<b>Level 4: Number Theory</b>	<b>89</b>
<b>Level 4: Algebra and Functions</b>	<b>95</b>
<b>Level 4: Geometry</b>	<b>100</b>
<b>Level 4: Probability and Statistics</b>	<b>109</b>
<b>Level 4: Trigonometry</b>	<b>113</b>
<b>Level 5: Number Theory</b>	<b>117</b>
<b>Level 5: Algebra and Functions</b>	<b>124</b>
<b>Level 5: Geometry</b>	<b>130</b>

<b>Level 5: Probability and Statistics</b>	139
<b>Level 5: Trigonometry</b>	143
<b>Supplemental Problems – Questions</b>	<b>147</b>
<b>Level 1: Number Theory</b>	147
<b>Level 1: Algebra and Functions</b>	149
<b>Level 1: Geometry</b>	150
<b>Level 1: Probability and Statistics</b>	150
<b>Level 2: Number Theory</b>	155
<b>Level 2: Algebra and Functions</b>	157
<b>Level 2: Geometry</b>	159
<b>Level 2: Probability and Statistics</b>	161
<b>Level 2: Trigonometry</b>	162
<b>Level 3: Number Theory</b>	164
<b>Level 3: Algebra and Functions</b>	166
<b>Level 3: Geometry</b>	168
<b>Level 3: Probability and Statistics</b>	170
<b>Level 3: Trigonometry</b>	172
<b>Level 4: Number Theory</b>	174
<b>Level 4: Algebra and Functions</b>	176
<b>Level 4: Geometry</b>	178
<b>Level 4: Probability and Statistics</b>	180
<b>Level 4: Trigonometry</b>	181
<b>Level 5: Number Theory</b>	183
<b>Level 5: Algebra and Functions</b>	185
<b>Level 5: Geometry</b>	187
<b>Level 5: Probability and Statistics</b>	191
<b>Level 5: Trigonometry</b>	193
<b>Answers to Supplemental Problems</b>	<b>195</b>
<b>Afterword: Your Road to Success</b>	<b>202</b>
<b><i>About the Author</i></b>	<b>204</b>
<b>Books By Dr. Steve Warner</b>	<b>205</b>

# I N T R O D U C T I O N

## THE PROPER WAY TO PREPARE



here are many ways that a student can prepare for the ACT. But not all preparation is created equal. I always teach my students the methods that will give them the maximum result with the minimum amount of effort.

The book you are now reading is self-contained. Each problem was carefully created to ensure that you are making the most effective use of your time while preparing for the ACT. By grouping the problems given here by level and topic I have ensured that you can focus on the types of problems that will be most effective to improving your score.

### 1. Using this book effectively

- Begin studying at least three months before the ACT
- Practice ACT math problems twenty minutes each day
- Choose a consistent study time and location

You will retain much more of what you study if you study in short bursts rather than if you try to tackle everything at once. So try to choose about a twenty minute block of time that you will dedicate to ACT math each day. Make it a habit. The results are well worth this small time commitment.

- Every time you get a question wrong, **mark it off, no matter what your mistake.**
- Begin each study session by first redoing problems from previous study sessions that you have marked off.
- If you get a problem wrong again, **keep it marked off.**

Note that this book often emphasizes solving each problem in more than one way. Please listen to this advice. The same question is not generally repeated on any ACT so the important thing is learning as many techniques as possible.

Being able to solve any specific problem is of minimal importance. The more ways you have to solve a single problem the more prepared you will be to tackle a problem you have never seen before, and the quicker you will be able to solve that problem. Also, if you have multiple methods for solving a single problem, then on the actual ACT when you “check over” your work you will be able to redo each problem in a different way. This will eliminate all “careless” errors on the actual exam. Note that in this book the quickest solution to any problem will always be marked with an asterisk (\*).

## 2. The magical mixture for success

A combination of three components will maximize your ACT math score with the least amount of effort.

- Learning test taking strategies that work specifically for standardized tests.
- Practicing ACT problems for a small amount of time each day for about three months before the ACT.
- Taking about four practice tests before test day to make sure you are applying the strategies effectively under timed conditions.

I will discuss each of these three components in a bit more detail.

**Strategy:** The more ACT specific strategies that you know the better off you will be. Throughout this book you will see many strategies being used. Some examples of basic strategies are “plugging in answer choices,” “taking guesses,” and “picking numbers.” Some more advanced strategies include “identifying arithmetic sequences with linear equations,” and “moving the sides of a figure around.” Pay careful attention to as many strategies as possible and try to internalize them. Even if you do not need to use a strategy for that specific problem, you will certainly find it useful for other problems in the future.

**Practice:** The problems given in this book are more than enough to vastly improve your current ACT math score. All you need to do is work on these problems for about ten to twenty minutes each day over a period of three to four months and the final result will far exceed your expectations.

Let me further break this component into two subcomponents – **topic** and **level**.

**Topic:** You want to practice each of the five general math topics given on the ACT and improve in each independently. The five topics are **Number Theory, Algebra and Functions, Geometry, Probability and Statistics, and Trigonometry**. The problem sets in this book are broken into these five topics.



**Level:** You will make the best use of your time by primarily practicing problems that are at and slightly above your current ability level. For example, if you are struggling with Level 2 Geometry problems, then it makes no sense at all to practice Level 5 Geometry problems. Keep working on Level 2 until you are comfortable, and then slowly move up to Level 3. Maybe you should never attempt those Level 5 problems. You can get an exceptional score without them (over a 30).

**Tests:** You want to take about four practice tests before test day to make sure that you are implementing strategies correctly and using your time wisely under pressure. For this task you should use actual ACT exams such as those found in the third edition of “The Real ACT Prep Guide.” Take one test every few weeks to make sure that you are implementing all the strategies you have learned correctly under timed conditions. Note that only the third edition has five actual ACTs.

### **3. Practice problems of the appropriate level**

In this book ACT math questions have been split into 5 Levels. Roughly speaking, the ACT math section increases in difficulty as you progress from question 1 to questions 60. So you can think of the first 12 problems as Level 1, the next 12 as Level 2 and so on.

Keep track of your current ability level so that you know the types of problems you should focus on. If you are currently scoring around a 15 on your practice tests, then you should be focusing primarily on Level 1, 2, and 3 problems. You can easily raise your score by 4 points without having to practice a single hard problem.

If you are currently scoring about a 20, then your primary focus should be Level 2 and 3, but you should also do some Level 1 and 4 problems.

If you are scoring around a 25, you should be focusing on Level 2, 3, and 4 problems, but you should do some Level 1 and 5 problems as well.

Those of you at the 30 level really need to focus on those Level 4 and 5 problems.

If you really want to refine your studying, then you should keep track of your ability level in each of the five major categories of problems:

- **Number Theory**
- **Algebra and Functions**
- **Probability, Statistics and Data Analysis**
- **Geometry**
- **Trigonometry**

For example, many students have trouble with very easy geometry problems, even though they can do more difficult number theory problems. This type of student may want to focus on Level 1, 2, and 3 geometry questions, but Level 3 and 4 number theory questions.

#### **4. Practice in small amounts over a long period of time**

Ideally you want to practice doing ACT math problems ten to twenty minutes each day beginning at least 3 months before the exam. You will retain much more of what you study if you study in short bursts than if you try to tackle everything at once.

The only exception is on a day you do a practice test. You should do at least four practice tests before you take the ACT. Ideally you should do your practice tests on a Saturday or Sunday morning. At first you can do just the math section. The last one or two times you take a practice test you should do the whole test in one sitting. As tedious as this is, it will prepare you for the amount of endurance that it will take to get through this exam.

So try to choose about a twenty minute block of time that you will dedicate to ACT math every night. Make it a habit. The results are well worth this small time commitment.

#### **5. Redo the problems you get wrong over and over and over until you get them right**

If you get a problem wrong, and never attempt the problem again, then it is extremely unlikely that you will get a similar problem correct if it appears on the ACT.

Most students will read an explanation of the solution, or have someone explain it to them, and then never look at the problem again. This is *not* how you optimize your ACT score. To be sure that you will get a similar problem correct on the ACT, you must get the problem correct before the ACT—and without actually remembering the problem.

This means that after getting a problem incorrect, you should go over and understand why you got it wrong, wait at least a few days, then attempt the same problem again. If you get it right you can cross it off your list of problems to review. If you get it wrong, keep revisiting it every few days until you get it right. Your score *does not* improve by getting problems correct. **Your score improves when you learn from your mistakes.**

## 6. Check your answers properly

When you go back to check your earlier answers for careless errors *do not* simply look over your work to try to catch a mistake. This is usually a waste of time. Always redo the problem without looking at any of your previous work. Ideally, you want to use a different method than you used the first time.

For example, if you solved the problem by picking numbers the first time, try to solve it algebraically the second time, or at the very least pick different numbers. If you do not know, or are not comfortable with a different method, then use the same method, but do the problem from the beginning and do not look at your original solution. If your two answers do not match up, then you know that this a problem you need to spend a little more time on to figure out where your error is.

This may seem time consuming, but that's ok. It is better to spend more time checking over a few problems than to rush through a lot of problems and repeat the same mistakes.

## 7. Take a guess whenever you cannot solve a problem

There is no guessing penalty on the ACT. Whenever you do not know how to solve a problem take a guess. Ideally you should eliminate as many answer choices as possible before taking your guess, but if you have no idea whatsoever do not waste time overthinking. Simply put down an answer and move on. You should certainly mark it off and come back to it later if you have time.

## 8. Pace yourself

Do not waste your time on a question that is too hard or will take too long. After you've been working on a question for about 30 to 45 seconds you need to make a decision. If you understand the question and think that you can get the answer in another 30 seconds or so, continue to work on the problem. If you still do not know how to do the problem or you are using a technique that is going to take a long time, mark it off and come back to it later if you have time.

If you do not know the correct answer, eliminate as many answer choices as you can and take a guess. But you still want to leave open the possibility of coming back to it later. Remember that every problem is worth the same amount. Do not sacrifice problems that you may be able to do by getting hung up on a problem that is too hard for you.

## 9. Attempt the right number of questions

Many students make the mistake of thinking that they have to attempt every single ACT math question when they are taking the test. There is no such rule. In fact, most students will increase their ACT score by *reducing* the number of questions they attempt. The following chart gives a general guideline for how many questions you should be attempting.

Score	Questions
< 13	15/60
13 – 14	20/60
15 – 17	28/60
18 – 20	36/60
21 – 23	40/60
24 – 27	52/60
28 – 36	60/60

For example, a student with a current score of 19 should be attempting about 36 of the 60 questions on the test.

Since the math questions on the ACT tend to start out easier in the beginning of the section and get harder as you go, then attempting the first 36 questions is not a bad idea. However, it is okay to skip several questions and try a few that appear later on.

Note that although the questions tend to get harder as you go, it is not true that each question is harder than the previous question. For example, it is possible for question 25 to be easier than question 24, and in fact, question 25 can even be easier than question 20. But it is unlikely that question 50 would be easier than question 20.

If you are particularly strong in a certain subject area, then you may want to “seek out” questions from that topic even though they may be more difficult. For example, if you are very strong at number theory problems, but very weak at probability problems, then you may want to try every number theory problem no matter where it appears, and you may want to reduce the number of probability problems you attempt.

Remember that there is no guessing penalty on the ACT, so you should *not* leave any questions blank. This *does not* mean you should attempt every question. It means that if you are running out of time make sure you fill in answers for all the questions you did not have time to attempt.

For example, if you are currently scoring a 21, then it is possible you will only be attempting the first 40 questions or so. Therefore when you are running out of time you should fill in answers for the last 20 problems. If you happen to get a chance to attempt some of them you can always change your answer. But make sure those answers are filled in before the test ends!

## 10. Use your calculator wisely.

- Use a TI-84 or comparable calculator if possible when practicing and during the ACT.
- Make sure that your calculator has fresh batteries on test day.
- Make sure your calculator is in degree mode. If you are using a TI-84 (or equivalent) calculator press **MODE** and on the third line make sure that **DEGREE** is highlighted. If it is not, scroll down and select it. If possible do not alter this setting until you are finished taking your ACT.

Below are the most important things you should practice on your graphing calculator.

- Practice entering complicated computations in a single step.
- Know when to insert parentheses:
  - Around numerators of fractions
  - Around denominators of fractions
  - Around exponents
  - Whenever you actually see parentheses in the expression

### Examples:

We will substitute a 5 in for  $x$  in each of the following examples.

<u>Expression</u>	<u>Calculator computation</u>
$\frac{7x+3}{2x-11}$	$(7*5 + 3)/(2*5 - 11)$
$(3x-8)^{2x-9}$	$(3*5 - 8)^(2*5 - 9)$

- Clear the screen before using it in a new problem. The big screen allows you to check over your computations easily.
- Press the **ANS** button (**2ND (-)**) to use your last answer in the next computation.
- Press **2ND ENTER** to bring up your last computation for editing. This is especially useful when you are plugging in answer choices, or guessing and checking.

- You can press **2ND ENTER** over and over again to cycle backwards through all the computations you have ever done.
- Know where the  $\sqrt{\quad}$ ,  $\pi$ ,  $\wedge$  and **LOG** buttons are so you can reach them quickly.
- Change a decimal to a fraction by pressing **MATH ENTER ENTER**.
- Press the **MATH** button - in the first menu that appears you can take cube roots and  $n$ th roots for any  $n$ . Scroll right to **NUM** and you have **lcm**( and **gcd**(. Scroll right to **PRB** and you have **nPr**, **nCr**, and **!** to compute permutations, combinations and factorials very quickly.
- Know how to use the **SIN**, **COS** and **TAN** buttons.

The following graphing tools can also be useful.

- Press the **Y=** button to enter a function, and then hit **ZOOM 6** to graph it in a standard window.
- Practice using the **WINDOW** button to adjust the viewing window of your graph.
- Practice using the **TRACE** button to move along the graph and look at some of the points plotted.
- Pressing **2ND TRACE** (which is really **CALC**) will bring up a menu of useful items. For example selecting **ZERO** will tell you where the graph hits the  $x$ -axis, or equivalently where the function is zero. Selecting **MINIMUM** or **MAXIMUM** can find the vertex of a parabola. Selecting **INTERSECT** will find the point of intersection of 2 graphs.

## PROBLEMS BY LEVEL AND TOPIC WITH FULLY EXPLAINED SOLUTIONS

**Note:** The quickest solution will always be marked with an asterisk (\*).

### LEVEL 1: NUMBER THEORY

1.  $|5(-4) + 3(5)| = ?$
- A.  $-35$
  - B.  $-5$
  - C.  $5$
  - D.  $35$
  - E.  $36$

Recall that  $|a|$  means the **absolute value** of  $a$ . It takes whatever number is between the two lines and makes it nonnegative. Here are a few examples:  $|3| = 3$ ,  $|-5| = 5$ ,  $|0| = 0$ .

\* **Quick solution:** Simply type the following into your calculator:

$$5(-4) + 3(5)$$

The output is  $-5$ . But we want the **absolute value** of this number. So the answer to the question is  $5$ , choice **C**.

**Remarks:** (1) You can use the absolute value function on your TI-84 calculator if you like. This can be found under the MATH menu. You would then type the following:

$$\text{abs}(5(-4) + 3(5))$$

The output will be  $5$ , choice **C**.

(2) Note that I left off the rightmost parenthesis in the computation in Remark (1) above. There is no need to close parentheses at the end of an expression. Your calculator will do it automatically.

**Solution by hand:**  $|5(-4) + 3(5)| = |-20 + 15| = |-5| = 5$ , choice **C**.

**Note:** In the hand solution, all multiplication was done first, followed by the addition. Finally, the absolute value was taken at the end.

**Order of Operations:** Here is a quick review of order of operations.

---

**PEMDAS**

<b>P</b>	Parentheses
<b>E</b>	Exponentiation
<b>M</b>	Multiplication
<b>D</b>	Division
<b>A</b>	Addition
<b>S</b>	Subtraction

---

Note that multiplication and division have the same priority, and addition and subtraction have the same priority.

2. The second term of an arithmetic sequence is 15 and the third term is 10. What is the first term?

**F.**  $-15$

**G.**  $-10$

**H.**  $\frac{1}{15}$

**J.**  $10$

**K.**  $20$

\* **Quick solution:** Moving backwards, to get from the third term to the second term we add 5. Therefore we add 5 more to get to the first term. So the first term is  $15 + 5 = 20$ , choice **K**.

**Remark:** In an arithmetic sequence, you always add (or subtract) the same number to get from one term to the next. This can be done by moving forwards or backwards through the sequence.

**For the advanced student:**

An **arithmetic sequence** is a sequence of numbers such that the difference  $d$  between consecutive terms is constant. The number  $d$  is called the **common difference** of the arithmetic sequence.

Here is an example of an arithmetic sequence: 20, 15, 10, 5, 0,  $-5$ ,  $-10$ , ...  
In this example the common difference is  $d = 15 - 20 = -5$ .

Note that this is the same arithmetic sequence given in the above ACT question.

**Arithmetic sequence formula:**  $a_n = a_1 + (n - 1)d$

In the above formula,  $a_n$  is the  $n$ th term of the sequence. For example,  $a_1$  is the first term of the sequence.



**Note:** In the arithmetic sequence 20, 15, 10, 5, 0, -5, -10, ... we have that  $a_1 = 20$  and  $d = -5$ . Therefore

$$a_n = 20 + (n - 1)(-5) = 20 - 5n + 5 = 25 - 5n.$$

It follows that  $a_1 = 25 - 5(1) = 25 - 5 = 20$ , choice **K**.

**Linear equations and arithmetic sequences:** Questions about arithmetic sequences can easily be thought of as questions about lines and linear equations. We can identify terms of the sequence with points on a line where the  $x$ -coordinate is the term number and the  $y$ -coordinate is the term itself.

**Remark:** In the question above, since the second term of the sequence is 15, we can identify this term with the point (2,15). Since the third term of the sequence is 10, we can identify this with the point (3,10). Note that the common difference  $d$  is just the slope of the line that passes through these two points, i.e.  $d = \frac{10-15}{3-2} = -5$ .

3. Joseph bought a tie for 60% of its original price of \$14.50 and a shirt for  $\frac{2}{5}$  of the original price of \$45.00. Ignoring sales tax, what is the total amount of these purchases?
- A. \$21.00
  - B. \$25.00
  - C. \$26.00
  - D. \$26.70
  - E. \$40.50

**\* Quick calculator computation:**

$$.60 * 14.50 + 2 / 5 * 45 = 26.70.$$

So the answer is choice **D**.

**Note:** (1) To change a percent to a decimal, divide by 100, or equivalently move the decimal point two places to the left (adding zeros if necessary). Note that the number 60 has an “invisible” decimal point after the 0 (so that  $60 = 60.$ ). Moving the decimal to the left two places gives us .60.

(2) The word “of” always translates to multiplication. So 60% of 14.50 is the same as  $.6(14.50)$ , and  $\frac{2}{5}$  of 45 is the same as  $\frac{2}{5} \cdot 45$ .

4. Dana needs  $5\frac{1}{12}$  ounces of a chemical for an experiment. She has  $3\frac{1}{4}$  ounces of the chemical. How many more ounces does she need?

F.  $1\frac{5}{12}$

G.  $1\frac{5}{6}$

H.  $2\frac{1}{6}$

J.  $2\frac{5}{6}$

K.  $2\frac{11}{12}$

**\* Quick calculator computation:** We type the following into our TI-84 calculator:  $5 + 1 / 12 - (3 + 1 / 4)$ .

The output is 1.83333333. So it looks like the answer is choice G. To be safe, let's type in our calculator  $1 + 5 / 6$ . The output is also 1.83333333.

So the answer is choice **G**.

5. A pack of 50 balloons is priced at \$3.50 now. If the balloons go on sale for 30% off the current price, what will be the sale price of the pack?

A. \$0.45

B. \$1.75

C. \$2.00

D. \$2.45

E. \$2.50

**\* Quick calculator computation:**

$$.7 * 3.50 = 2.45$$

So the answer is choice **D**.

**Remark:** (1) "30% off" is the same as "70% of." So we are taking 70% of 3.50. As in problem 3 above, we change 70% to a decimal by moving the "invisible" decimal point to the left 2 places to get .7.

(2) We can also take 30 percent off of 3.50 by taking 30 percent of 3.50 and then subtracting this from 3.50.

So 30 percent of 3.50 is  $.3(3.50) = 1.05$ . Therefore 30 percent off of 3.50 is  $3.50 - 1.05 = 2.45$ , choice **D**.

6. Which of the following lists all the positive factors of 27 ?

- F. 1, 27
- G. 3, 9
- H. 3, 9, 27
- J. 27, 54, 81
- K. 1, 3, 9, 27

\* Since  $(3)(9) = 27$ , 3 and 9 are both factors of 27. Also,  $(1)(27) = 27$ . So 1 and 27 are also factors of 27. So the answer is choice **K**.

**Remark:** It is easy to check whether one integer is a factor of another integer with our calculator. For example if we divide 27 by 3 in our calculator we get 9. Since 9 is an integer, 3 is a factor of 27.

**Definitions:** The **integers** are the counting numbers together with their negatives.

$$\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$$

The **positive integers** consist of the positive numbers from that set.

$$\{1, 2, 3, 4, \dots\}$$

An integer  $d$  is a **factor** of another integer  $n$  if there is an integer  $k$  such that  $n = dk$ . For example 3 is a factor of 27 because  $27 = (3)(9)$ .

7. What is the largest integer less than  $\sqrt{73}$  ?

- A. 3
- B. 5
- C. 7
- D. 8
- E. 9

**Solution by plugging in answer choices:** Since the word “largest” appears in the problem, let’s start with the largest answer choice, choice E. We have  $9^2 = 81$ . This is too big. So let’s try choice D. Since  $8^2 = 64$  and  $64 < 73$ , we see that the answer is choice **D**.

\* **Quick solution:** If we take the square root of 73 in our calculator we get approximately 8.544. The largest integer less than this is 8, choice **D**.

8. Philip earns \$9.00 per hour for up to 40 hours of work in a week. For each hour over 40 hours of work in a week, Philip earns twice his regular pay. How much does Philip earn for a week in which he works 43 hours?
- F. \$387.00
  - G. \$400.50
  - H. \$414.00
  - J. \$472.50
  - K. \$512.00

\* **Quick calculator computation:**

$$9(40) + 18(3) = 414.$$

So Philip earns \$414.00, choice **H**.

## LEVEL 1: ALGEBRA AND FUNCTIONS

9. If  $a = -5$ , what is the value of  $\frac{a^2-4}{a+2}$  ?
- A.  $-7$
  - B.  $-4$
  - C.  $4$
  - D.  $9\frac{2}{3}$
  - E.  $12$

We have  $a^2 - 4 = (-5)^2 - 4 = 25 - 4 = 21$ . Also  $a + 2 = -5 + 2 = -3$ . Finally we divide  $21 / (-3) = -7$ , choice **A**.

**Remarks:** (1) If you are doing these computations on your calculator, make sure that  $-5$  is put in parentheses before squaring:  $-5 \wedge 2$  will give an output of  $-25$  which is not correct.

(2) This can be done with a single calculator computation as follows:

$$((-5) \wedge 2 - 4) / (-5 + 2)$$

Note that the whole numerator is inside parentheses and the whole denominator is inside parentheses.

10.  $x^2 - 73x + 27 - 46x^2 + 75x$  is equivalent to:

- F.  $-29x^2$
- G.  $-29x^6$
- H.  $-45x^4 + 2x^2 + 27$
- J.  $-45x^2 + 2x + 27$
- K.  $-44x^2 + 2x + 27$

**Solution by picking a number:** Let's choose a value for  $x$ , say  $x = 2$ .  
Then

$$x^2 - 73x + 27 - 46x^2 + 75x = 2^2 - 73(2) + 27 - 46(2)^2 + 75(2) = -149$$

Put a nice big dark circle around  $-149$  so you can find it easier later. We now substitute 2 for  $x$  into each answer choice:

- F.  $-29(2)^2 = -116$
- G.  $-29(2)^6 = -1856$
- H.  $-45(2)^4 + 2(2)^2 + 27 = -685$
- J.  $-45x^2 + 2x + 27 = -149$
- K.  $-44x^2 + 2x + 27 = -145$

Since F, G, H, and K each came out incorrect, the answer is choice **J**.

**Important note:** J is **not** the correct answer simply because it is equal to  $-149$ . It is correct because all four of the other choices are **not**  $-149$ .  
**You absolutely must check all five choices!**

\* **Algebraic solution:**

$$\begin{aligned} x^2 - 73x + 27 - 46x^2 + 75x &= x^2 - 46x^2 - 73x + 75x + 27 \\ &= (1 - 46)x^2 + (-73 + 75)x + 27 = -45x^2 + 2x + 27 \end{aligned}$$

This is choice **J**.

11. If  $4b - 5 = 17$ , then  $b =$

- A. 4.0
- B. 5.5
- C. 10.0
- D. 17.5
- E. 22.0

**Solution by plugging in answer choices:** Let's start with choice C and guess that  $b = 10$ . Then  $4b - 5 = 4(10) - 5 = 40 - 5 = 35$ . This is too big. So we can eliminate choices C, D, and E.

Let's try choice B next. So we are guessing that  $b = 5.5$ . We then have that  $4b - 5 = 4(5.5) - 5 = 22 - 5 = 17$ . This is correct. So the answer is choice **B**.

\* **Algebraic solution:** We add 5 to each side of the equation  $4b - 5 = 17$  to get  $4b = 22$ . We then divide each side of this equation by 4 to get that  $b = 5.5$ , choice **B**.

12. Which of the following is an equivalent simplified expression for  $3(5x + 8) - 4(3x - 2)$  ?

- F.  $x + 16$
- G.  $3x + 16$
- H.  $3x + 32$
- J.  $7x + 2$
- K.  $7x + 3$

**Solution by picking a number:** Let's choose a value for  $x$ , say  $x = 2$ . Then

$$\begin{aligned} 3(5x + 8) - 4(3x - 2) &= 3(5 \cdot 2 + 8) - 4(3 \cdot 2 - 2) = 3(10 + 8) - 4(6 - 2) \\ &= 3(18) - 4(4) = 54 - 16 = \mathbf{38}. \end{aligned}$$

Put a nice big dark circle around **38** so you can find it easier later. We now substitute 2 for  $x$  into each answer choice.

- F. 18
- G. 22
- H. 38
- J. 16
- K. 17

Since F, G, J, and K each came out incorrect, the answer is choice **H**.

**Important note:** H is **not** the correct answer simply because it is equal to 38. It is correct because all four of the other choices are **not** 38. **You absolutely must check all five choices!**

**Remark:** The computation  $3(5 \cdot 2 + 8) - 4(3 \cdot 2 - 2)$  can be done in a single step with your calculator. Simply input the following.

$$3(5 * 2 + 8) - 4(3 * 2 - 2) \text{ ENTER}$$

**\* Algebraic solution:**

$$3(5x + 8) - 4(3x - 2) = 15x + 24 - 12x + 8 = 3x + 32$$

This is choice **H**.

**Note:** Make sure you are using the distributive property correctly here. For example  $3(5x + 8) = 15x + 24$ . A common mistake would be to write  $3(5x + 8) = 15x + 8$ .

13. A Celsius temperature  $C$  can be approximated by subtracting 32 from the Fahrenheit temperature  $F$  and then multiplying by  $\frac{1}{2}$ . Which of the following expresses this approximation method? (Note: The symbol  $\approx$  means “is approximately equal to.”)

A.  $C \approx \frac{1}{2}(F - 32)$

B.  $C \approx \frac{1}{2}F - 32$

C.  $C \approx 2(F - 32)$

D.  $C \approx 2F - 32$

E.  $C \approx \sqrt{F} - 32$

**\* When we subtract 32 from  $F$  we get  $F - 32$ . Multiplying this expression by  $\frac{1}{2}$  yields  $\frac{1}{2}(F - 32)$ . This is choice **A**.**

**Caution:** A common mistake would be to multiply only the first term by  $\frac{1}{2}$  to get  $\frac{1}{2}F - 32$ . This is wrong. Whenever you are doing anything to an expression always keep it in parentheses as was done in the solution above.

14. Which of the following expressions is equivalent to  $5a + 10b + 15c$  ?
- F.  $5(a + 2b + 3c)$
- G.  $5(a + 2b + 15c)$
- H.  $5(a + 10b + 15c)$
- J.  $5(a + 2b) + 3c$
- K.  $30(a + b + c)$

**Solution by picking numbers:** Let's choose values for  $a$ ,  $b$ , and  $c$ , say  $a = 2$ ,  $b = 3$ ,  $c = 4$ . Then

$$5a + 10b + 15c = 5(2) + 10(3) + 15(4) = 10 + 30 + 60 = \mathbf{100}.$$

Put a nice big dark circle around **100** so you can find it easier later. We now substitute  $a = 2$ ,  $b = 3$ ,  $c = 4$  into each answer choice:

$$\mathbf{F.} \quad 5(2 + 2 \cdot 3 + 3 \cdot 4) = 100$$

$$\mathbf{G.} \quad 5(2 + 2 \cdot 3 + 15 \cdot 4) = 340$$

$$\mathbf{H.} \quad 5(2 + 10 \cdot 3 + 15 \cdot 4) = 460$$

$$\mathbf{J.} \quad 5(2 + 2 \cdot 3) + 3 \cdot 4 = 52$$

$$\mathbf{K.} \quad 30(2 + 3 + 4) = 270$$

Since G, H, J, and K each came out incorrect, the answer is choice **F**.

**Important note:** F is **not** the correct answer simply because it is equal to 100. It is correct because all four of the other choices are **not** 100. **You absolutely must check all five choices!**

**Remark:** All of the above computations can be done in a single step with your calculator.

\* **Algebraic solution:** We simply factor out a 5 to get

$$5a + 10b + 15c = 5(a + 2b + 3c)$$

This is choice **F**.

**Remarks:** (1) If you have trouble seeing why the right hand side is the same as what we started with on the left, try working backwards and multiplying instead of factoring. In other words we have

$$5(a + 2b + 3c) = 5a + 10b + 15c$$

Note how the distributive property is being used here. Each term in parentheses is multiplied by the 5!

(2) You can also start with the answer choices and do each multiplication (as we did in Remark (1)) until you get  $5a + 10b + 15c$ .

15. Which of the following is a value for  $z$  that solves the equation  $|z - 4| = 9$ ?

A.  $-13$

B.  $-5$

C.  $\frac{9}{4}$

D.  $5$

E.  $36$



**Solution by plugging in answer choices:** Normally I would start with choice C, but in this case there are much simpler numbers to plug in, so let's start with choice B and guess that  $z = -5$ . Then we have

$$|z - 4| = |-5 - 4| = |-9| = 9.$$

This is correct. So the answer is choice **B**.

\* **Algebraic solution:** The absolute value equation  $|z - 4| = 9$  is equivalent to the **two** equations

$$z - 4 = 9$$

$$z - 4 = -9$$

Adding 4 to each side of each of these equations yields  $z = 9 + 4 = 13$  or  $z = -9 + 4 = -5$ . Only the latter answer is an answer choice. So the answer is choice **B**.

16. If  $c > 1$ , then which of the following has the least value?

- F.  $\sqrt{c}$
- G.  $\sqrt{2c}$
- H.  $\sqrt{c^2}$
- J.  $c\sqrt{c}$
- K.  $c^2$

**Solution by picking a number:** Let's choose a value for  $c$  that is greater than 1, say  $c = 2$ , and plug this value into each answer choice (using our calculator to get an approximate answer).

- F.  $\sqrt{2} \approx 1.414$
- G.  $\sqrt{2 \cdot 2} = 2$
- H.  $\sqrt{2^2} = 2$
- J.  $2\sqrt{2} \approx 2.828$
- K.  $2^2 = 4$

We see that the least value appears in choice **F**.

**LEVEL 1: GEOMETRY**

17. In  $\triangle PQR$ , the sum of the measures of  $\angle P$  and  $\angle Q$  is  $59^\circ$ . What is the measure of  $\angle R$  ?
- A.  $31^\circ$
  - B.  $59^\circ$
  - C.  $118^\circ$
  - D.  $121^\circ$
  - E.  $131^\circ$

**Solution by plugging in answer choices:** First note that the three angle measures in a triangle add up to  $180^\circ$ .

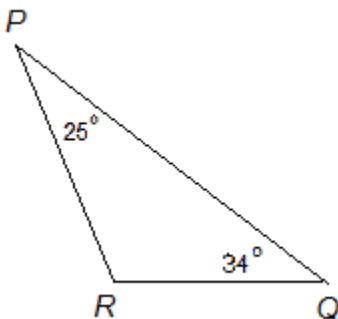
Let's start with choice C and guess that the measure of angle  $R$  is  $118^\circ$ . We have  $59 + 118 = 177$ . This is a bit too small. So we can eliminate choices A, B, and C.

Let's try choice D next and guess that the measure of angle  $R$  is  $121^\circ$ . We have  $59 + 121 = 180$ . This is correct. So the answer is **D**.

\* **Quick solution:** Since the three angle measures in a triangle add up to 180 degrees, the measure of angle  $R$  is  $180 - 59 = 121^\circ$ , choice **D**.

**Note:**  $59^\circ$  is the sum of two angle measures. So adding this to the third angle measure gives  $180^\circ$ .

**Solution by drawing a picture and picking numbers:** Let's draw a picture.



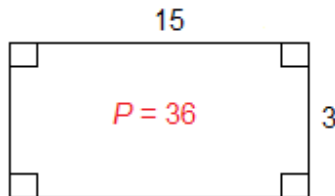
In the above picture I chose angle  $P$  and angle  $Q$  to have measures that add to 59 degrees. It follows that the measure of angle  $R$ , in degrees is  $180 - 25 - 34 = 121$ , choice **D**.

**Definition:** A **triangle** is a two-dimensional geometric figure with three sides and three angles. The sum of the degree measures of all three angles of a triangle is  $180^\circ$ .

18. What is the perimeter, in centimeters, of a rectangle with length 15 in and width 3 in?
- F. 18
  - G. 21
  - H. 36
  - J. 45
  - K. 90

\* **Quick solution:**  $P = 2\ell + 2w = 2(15) + 2(3) = 30 + 6 = 36$ , choice **H**.

Here is a picture for extra clarification.



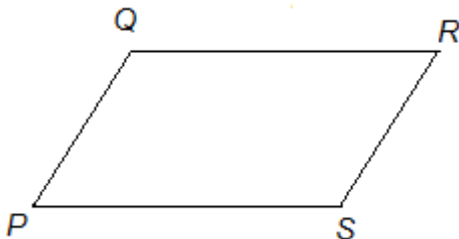
**Definitions:** A **quadrilateral** is a two-dimensional geometric figure with four sides and four angles. The sum of the degree measures of all four angles of a quadrilateral is 360.

A **rectangle** is a quadrilateral in which each angle is a right angle. That is, each angle measures  $90^\circ$ .

The **perimeter** of a rectangle is  $P = 2\ell + 2w$ .

19. In parallelogram  $PQRS$ , which of the following must be true about the measures of  $\angle PQR$  and  $\angle QRS$  ?
- A. each are  $90^\circ$
  - B. each are less than  $90^\circ$
  - C. each are greater than  $90^\circ$
  - D. they add up to  $90^\circ$
  - E. they add up to  $180^\circ$

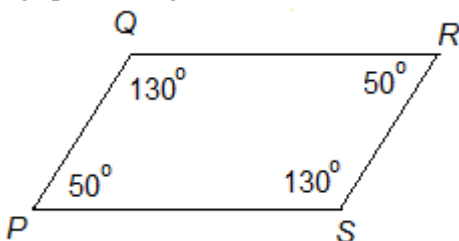
**Solution by drawing a picture and process of elimination:** Let's draw a picture.



Note that the measure of angle  $PQR$  is greater than 90 degrees, and the measure of angle  $QRS$  is less than 90 degrees. So we can eliminate choices A, B, and C.

Since the measure of angle  $PQR$  by itself is greater than 90 degrees, we can eliminate choice D. Therefore the answer is choice **E**.

**Note:** If you have trouble eliminating choices from the general picture above, try choosing specific angle measures. Here is an example:



Observe that we had to choose angle measures that sum to 360 (since  $PQRS$  is a quadrilateral). We also had to make sure that opposite angles were congruent (since  $PQRS$  is a parallelogram).

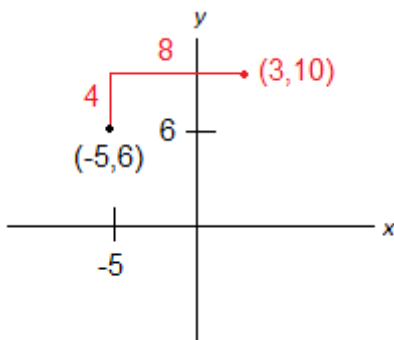
\* **Quick solution:** If you happen to recall that adjacent angles of a parallelogram are **supplementary** (have measures which add to 180 degrees), you could choose choice **E** immediately (or after drawing the first picture above, if necessary).

#### **Facts about parallelograms:**

- (1) opposite sides are congruent
- (2) opposite sides are parallel
- (3) opposite angles are congruent
- (4) the diagonals bisect each other

20. A point at  $(-5,6)$  in the standard  $(x,y)$  coordinate plane is shifted up 4 units and right 8 units. What are the coordinates of the new point?
- F.  $(-1,14)$   
G.  $(-13,10)$   
H.  $(-13, 2)$   
J.  $( 3, 2)$   
K.  $( 3,10)$

**Solution by drawing a picture:** Let's draw a picture of this situation.



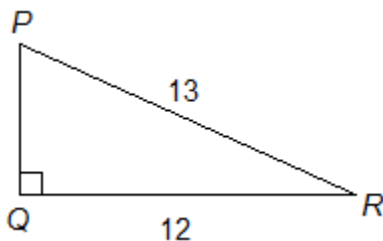
Note that to plot the point  $(-5,6)$ , from the origin  $(0,0)$  we move left 5 and up 6. From there we move up 4 and right 8 to get to the point  $(3,10)$ , choice **K**.

**Note:** We are adding 4 to the  $y$ -coordinate of the point and 8 to the  $x$ -coordinate of the point:  $6 + 4 = 10$  and  $-5 + 8 = 3$ .

21. The interior dimensions of a rectangular box are 5 inches by 4 inches by 3 inches. What is the volume, in cubic inches, of the interior of the box?
- A. 12  
B. 60  
C. 90  
D. 120  
E. 124

The volume of the box, in inches, is simply  $V = \ell wh = (5)(4)(3) = 60$ , choice **B**.

22. Given right triangle  $\triangle PQR$  below, what is the length of  $\overline{PQ}$  ?



- F.  $\sqrt{2}$
- G.  $\sqrt{5}$
- H. 5
- J. 7
- K. 11

**\* Solution using Pythagorean triples:** We use the Pythagorean triple 5, 12, 13 to see that  $PQ = 5$ , choice **H**.

**Note:** The most common Pythagorean triples are 3,4,5 and 5, 12, 13. Two others that may come up are 8, 15, 17 and 7, 24, 25.

**Solution by the Pythagorean Theorem:** By the Pythagorean Theorem, we have  $13^2 = (PQ)^2 + 12^2$ . So  $169 = (PQ)^2 + 144$ . Subtracting 144 from each side of this equation yields  $25 = (PQ)^2$ , or  $PQ = 5$ , choice **H**.

**Remarks:** (1) The Pythagorean Theorem says that if a right triangle has legs of length  $a$  and  $b$ , and a hypotenuse of length  $c$ , then  $c^2 = a^2 + b^2$ .

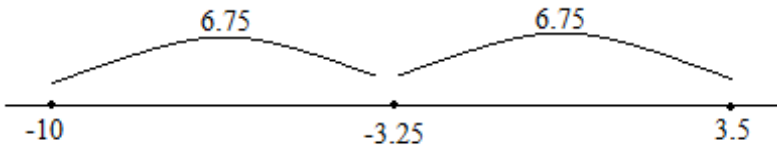
(2) Be careful in this problem: the length of the hypotenuse is 13. So we replace  $c$  by 13 in the Pythagorean Theorem

(3) The equation  $x^2 = 25$  would normally have two solutions:  $x = 5$  and  $x = -5$ . But the length of a side of a triangle cannot be negative, so we reject  $-5$ .

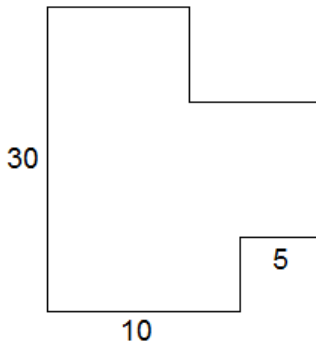
23. On a real number line, point  $X$  is at  $-3.25$  and is 6.75 units from point  $Y$ . What are the possible locations of point  $Y$  on the real number line?
- A.  $-10$  and  $-3.5$
  - B.  $-10$  and  $3.5$
  - C.  $-10$  and  $10$
  - D.  $10$  and  $-3.5$
  - E.  $10$  and  $3.5$

\* We compute  $-3.25 + 6.75 = 3.5$  and  $-3.25 - 6.75 = -10$ , choice **B**.

Here is a picture illustrating this problem.



24. In the figure below, adjacent sides meet at right angles and the lengths given are in inches. What is the perimeter of the figure, in inches?



- F. 45
- G. 60
- H. 90
- J. 120
- K. 450

\* **Solution by moving the sides of the figure around:** Recall that to compute the perimeter of the figure we need to add up the lengths of all 8 line segments in the figure. We “move” the two smaller vertical segments to the right, and each of the smaller horizontal segments up or down as shown below.



Note that the “bold” length is equal to the “dashed” length. We get a rectangle with length 30 and width 15. Thus, the perimeter is

$$(2)(30) + (2)(15) = 60 + 30 = 90.$$

This is choice **H**.

**Warning:** Although lengths remain unchanged by moving line segments around, areas will be changed. This method should **not** be used in problems involving areas.

## LEVEL 1: PROBABILITY AND STATISTICS

25. Jason has taken 4 tests in his math class, with grades of 73, 86, 64, and 97. In order to maintain this exact average, what *must* be Jason’s grade on his 5th math test?

- A. 60
- B. 70
- C. 80
- D. 90
- E. 95

Jason’s average on the 4 tests is  $\frac{73+86+64+97}{4} = 80$ . In order to maintain this exact average, Jason must receive an 80 on his 5th test, choice **C**.

**Notes:** (1) The **average (arithmetic mean)** of a list of numbers is the sum of the numbers in the list divided by the quantity of the numbers in the list.

$$\text{Average} = \frac{\text{Sum}}{\text{Number}}$$

(2) When we add a new number to a list of numbers, the average will remain the same precisely when the new number is equal to the old average.

If the new number is greater than the old average, the average will increase, and if the new number is less than the old average, the average will decrease.



26. A menu lists 2 appetizers, 5 meals, 4 drinks, and 3 desserts. A dinner consists of 1 of each of these 4 items. How many different dinners are possible from this menu?
- F. 2
  - G. 4
  - H. 14
  - J. 72
  - K. 120

**\* Solution using the counting principle:**  $(2)(5)(4)(3) = 120$ , choice **K**.

**Remark:** The **counting principle** says that if one event is followed by a second independent event, the number of possibilities is multiplied.

More generally, if  $E_1, E_2, \dots, E_n$  are  $n$  independent events with  $m_1, m_2, \dots, m_n$  possibilities, respectively, then event  $E_1$  followed by event  $E_2$ , followed by event  $E_3, \dots$ , followed by event  $E_n$  has  $m_1 \cdot m_2 \cdots m_n$  possibilities.

In this question there are 4 events: “choosing an appetizer,” “choosing a meal,” “choosing a drink,” and “choosing a dessert.”

27. In an urn with 60 marbles, 20% of the marbles are black. If you randomly choose a marble from the urn, what is the probability that the marble chosen is not one of the black marbles?
- A.  $\frac{1}{2}$
  - B.  $\frac{1}{5}$
  - C.  $\frac{2}{5}$
  - D.  $\frac{3}{5}$
  - E.  $\frac{4}{5}$

**\* 20% of 60 is  $(0.2)(60) = 12$ . So 12 of the marbles are black. Therefore  $60 - 12 = 48$  of the marbles are not black. So the requested probability is  $\frac{48}{60} = \frac{4}{5}$ , choice **E**.**

**Remarks:** (1) To compute a simple probability where all outcomes are equally likely, divide the number of “successes” by the total number of outcomes.

In this problem, the total is 60 marbles, and 48 of them are “successes.”

(2) You can quickly reduce a fraction in your TI-84 calculator by performing the division and then pressing MATH ENTER ENTER.

**Alternate solution:** As in the first solution we see that 12 of the marbles are black. So the probability of choosing a black marble is  $\frac{12}{60} = \frac{1}{5}$ . The probability of choosing a marble that is not black is  $1 - \frac{1}{5} = \frac{4}{5}$ , choice **E**.

**Note:** If  $P(E)$  is the probability of event  $E$  occurring, then the probability of event  $E$  not occurring is  $1 - P(E)$ .

28. A data set contains 6 elements and has a mean of 5. Five of the elements are 2, 4, 6, 8, and 10. Which of the following is the sixth element?

- F. 0
- G. 1
- H. 2
- J. 3
- K. 4

\* **Solution by changing the average to a sum:** We change the average (or mean) to a sum using the formula

$$\text{Sum} = \text{Average} \cdot \text{Number}$$

Here we are averaging 6 elements. Thus, the **Number** is 6. The **Average** is given to be 5. Therefore the **Sum** of the 6 numbers is  $5 \cdot 6 = 30$ . The sixth element is therefore

$$30 - 2 - 4 - 6 - 8 - 10 = 0$$

This is choice **A**.

29. If the probability that it will be sunny tomorrow is 0.4, what is the probability that it will not be sunny tomorrow?
- A. 1.4
  - B. 1.0
  - C. 0.6
  - D. 0.1
  - E. 0.0

\* The requested probability is  $1 - 0.4 = 0.6$ , choice **C**.

See the note at the end of question 27 for further explanation.

30. The average of 8 numbers is 7.3. If each of the numbers is decreased by 6, what is the average of the 8 new numbers?
- F. 0.0
  - G. 0.3
  - H. 1.3
  - J. 2.3
  - K. 7.3

**Solution by changing the average to a sum:** We change the average (or mean) to a sum using the formula

$$\text{Sum} = \text{Average} \cdot \text{Number}$$

Here we are averaging 8 numbers. Thus the **Number** is 8. The **Average** is given to be 7.3. Therefore the **Sum** of the 8 numbers is  $7.3 \cdot 8 = 58.4$ .

Since each number is decreased by 6, the sum is decreased by  $6 \cdot 8 = 48$ . So the sum of the 8 new numbers is  $58.4 - 48 = 10.4$ , and the average of the 8 new numbers is  $\frac{10.4}{8} = 1.3$ , choice **H**.

\* **Quick solution:** Since each number is decreased by the same amount, the average is decreased by this amount as well. So the average of the 8 new numbers is  $7.3 - 6 = 1.3$ , choice **H**.

31. Markus has 4 white hats and 5 black hats in his closet. If he randomly takes 1 of these 9 hats from his closet, what is the probability that the hat that Markus takes is black?
- A.  $\frac{1}{9}$
  - B.  $\frac{1}{5}$
  - C.  $\frac{4}{9}$
  - D.  $\frac{5}{9}$
  - E.  $\frac{5}{4}$

\* Recall from the first Remark in problem 27 that to compute a simple probability where all outcomes are equally likely, we divide the number of “successes” by the total number of outcomes.

In this problem, the total is 9 hats, and 5 of them are “successes.” So the probability that the hat Markus takes is black is  $\frac{5}{9}$ , choice **D**.

32. Debra’s grades on the first 3 tests in her math class were 86, 74, and 93. How many points must Debra receive on the fourth test to average exactly 85 points for these 4 tests (assume that all 4 tests are equally weighted)?
- F. 86
  - G. 87
  - H. 88
  - J. 89
  - K. 90

**Solution by changing the average to a sum:** We change the given average to a sum using the formula

$$\text{Sum} = \text{Average} \cdot \text{Number}$$

Here we are averaging 4 test grades. Thus, the **Number** is 4. The **Average** of the 4 test grades is given to be 85. Therefore the **Sum** of the 4 test grades is  $85 \cdot 4 = 340$ . So the number of points Debra must receive on the fourth test is

$$340 - 86 - 74 - 93 = 87$$

This is choice **G**.

This is the end of this free sample. To order this book from Amazon click the following image.

