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By Dr. Steve Warner

For the Revised SAT March 2016 and Beyond

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Problems by Level and Topic with Fully Explained Solutions

Note: An asterisk (*) before a question indicates that a calculator is required. An asterisk (*) before a solution indicates that the quickest solution is being given.

Level 1: Heart of Algebra

1. An author has a book available in paperback and digital formats. The author earns $2.47 on each paperback sale and $3.56 for each digital download. Which of the following expressions represents the amount, in dollars, that the author earns if p paperbacks are sold and d digital books are downloaded?

   (A) $2.47p + 3.56d$
   (B) $2.47p - 3.56d$
   (C) $3.56p + 2.47d$
   (D) $3.56p - 2.47d$

*** Algebraic solution:** The total amount the author earns from paperback sales, in dollars, is $2.47p$ and the total amount the author earns in digital downloads is $3.56d$. So all together the total amount that the author earns is $2.47p + 3.56d$, choice A.

Notes: (1) If 1 paperback is sold, the author earns 2.47 dollars.

If 2 paperbacks are sold, the author earns $2.47 \cdot 2 = 4.94$ dollars.

If 3 paperbacks are sold, the author earns $2.47 \cdot 3 = 7.41$ dollars.

Following this pattern, we see that if p paperbacks are sold, the author earns $2.47p$ dollars.

(2) A similar analysis to what was done in Note (1) shows that if d digital books are downloaded, the author earns $3.56d$ dollars. Try plugging in different values for d, starting at $d = 1$, so that you can see this for yourself.

Solution by picking numbers: Let's suppose that 10 paperbacks were sold and the book was downloaded 2 times. Then we have $p = 10$ and $d = 2$.

The total amount the author earns from paperback sales, in dollars, is $2.47 \cdot 10 = 24.70$. The total amount the author earns from digital downloads is $3.56 \cdot 2 = 7.12$ So the total the author earns, in dollars, is $24.70 + 7.12 = \mathbf{31.82}$.

Put a nice big dark circle around $\mathbf{31.82}$ so you can find it easier later. We now substitute $p = 10$ and $d = 2$ into each answer choice:

(A) $24.7 + 7.12 = 31.82$
(B) $24.7 - 7.12 = 17.58$
(C) $35.6 + 4.94 = 40.54$
(D) $35.6 - 4.94 = 30.66$

Since B, C and D each came out incorrect, the answer is choice A**.**

Important note: A is **not** the correct answer simply because it is equal to 31.82. It is correct because all three of the other choices are **not** 31.82. **You absolutely must check all four choices!**

Remark: All of the above computations can be done in a single step with your calculator (if a calculator is allowed for this problem).

Notes about picking numbers: (1) Observe that we picked a different number for each variable. We are less likely to get more than one answer choice to come out to the correct answer this way.

(2) We picked numbers that were simple, but not too simple. In general we might want to avoid 0 and 1 because more than one choice is likely to come out correct with these choices. 2 and 3 would normally be good choices (especially if a calculator is allowed). In this case 10 is particularly nice because multiplying by 10 is very easy (just move the decimal point to the right one unit).

(3) When using the strategy of picking numbers, it is very important that we check every answer choice. It is possible for more than one choice to come out to the correct answer. We would then need to pick new numbers to try to eliminate all but one choice.

2. If $5b + 3 < 18$, which of the following CANNOT be the value of b ?

(A) 0
(B) 1
(C) 2
(D) 3

Solution by starting with choice D: We start with choice D and substitute 3 in for b in the given inequality.

$$5b + 3 < 18$$
$$5(3) + 3 < 18$$
$$15 + 3 < 18$$
$$18 < 18$$

Since this is FALSE, the answer is choice D.

Notes: (1) A basic SAT math strategy that every student should know is "plugging in the answer choices." To use this strategy, we simply try out each answer choice until we find the one that "works." If we have no other information we would generally start with choice B or C as our first guess. In this particular problem, a little thought should convince you that the answer must be one of the extreme values.

(2) If we were to try choice C first, then the left hand side of the inequality gives us $5(2) + 3 = 10 + 3 = 13$. Since $13 < 18$ is true, we see that 2 CAN be a solution. This computation not only allows us to eliminate choice C as an answer, but choices A and B as well.

***** (3) A moment's thought tells us that we are looking for a number that is too big. So the largest number given must be the answer. Using this reasoning, we can actually solve this problem without doing a single computation.

Algebraic solution:

$$5b + 3 < 18$$
$$5b < 15$$
$$b < 3$$

Thus, the answer is choice D.

Notes: (1) We get from the first inequality to the second by subtracting 3 from each side: $(5b + 3) - 3 = 5b$ and $18 - 3 = 15$

(2) We get from the second inequality to the third inequality by dividing each side by 5: $\frac{5b}{5} = b$ and $\frac{15}{5} = 3$.

3. Perfect Floors Carpeting is hired to lay down carpet in k rooms of equal size. Perfect Floor's fee can be calculated by the expression $kA\ell w$, where k is the number of rooms, A is the cost per square foot of the carpet in dollars, ℓ is the length of each room in feet, and w is the width of each room in feet. If the customer chooses to use more expensive material for the carpet, which of the factors in the expression would change?

(A) k
(B) A
(C) ℓ
(D) w

* A is the cost per square foot of the carpet in dollars. If a customer chooses to use more expensive carpeting material, then A will increase. So the answer is B.

Notes: (1) k would increase if the customer decides to have more rooms carpeted (the rooms would need to be of equal size), and k would decrease if the customer decided to carpet less rooms.

(2) ℓ would increase if the customer decided to carpet rooms that were longer, and ℓ would decrease if the customer decided to carpet rooms that were shorter.

(3) w would increase if the customer decided to carpet rooms that were wider, and w would decrease if the customer decided to carpet rooms that were less wide.

4. For $i = \sqrt{-1}$, the sum $(-2 + 7i) + (-3 - 4i)$ is equal to

(A) $-5 - 11i$
(B) $-5 + 3i$
(C) $1 - 11i$
(D) $1 + 3i$

* $(-2 + 7i) + (-3 - 4i) = (-2 - 3) + (7 - 4)i = -5 + 3i$, choice B.

Notes: (1) The numbers $-2 + 7i$ and $-3 - 4i$ are **complex numbers**. In general, a complex number has the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$.

a is called the **real part** of the complex number and b is called the **imaginary part** of the complex number.

(2) We add two complex numbers simply by adding their real parts, and then adding their imaginary parts.

$$(a+bi)+(c+di)=(a+c)+(b+d)i$$

In this question, we have $a=-2$, $b=7$, $c=-3$, and $d=-4$.

5. If $c>0$ and $5c^2-45=0$, what is the value of c ?

*** Algebraic solution:** We add 45 to each side of the equation to get $5c^2=45$. We then divide each side of this last equation by 5 to get $c^2=\frac{45}{5}=9$. Since $3^2=9$ and $3>0$, the answer is $\mathbf{3}$.

Notes: (1) We can also begin by factoring and dividing each side of the given equation by 5.

$$5(c^2-9)=0$$
$$c^2-9=0$$

We can then add 9 to each side of the equation to get $c^2=9$, and then proceed as in the solution above to get $c=3$.

(2) The equation $c^2=9$ has two solution $c=\pm 3$. We reject the negative solution because we are given $c>0$.

(3) As an alternative to the method in Note (1), we can factor c^2-9 as the difference of two squares: $c^2-9=(c-3)(c+3)$, and then set each factor equal to 0 to get $c-3=0$ or $c+3=0$. From these last two equations we get $c=3$ or $c=-3$. Once again, since we are given $c>0$, we choose $c=3$.

(4) We can also just plug guesses for c in to the left hand side of the equation until we get 0. We will eventually see that

$$5(3)^2-45=5\cdot 9-45=45-45=0.$$

So the answer is 3.

6. If $2\left(\frac{x-3}{5}\right)=b$ and $b=4$, what is the value of x ?

*** Algebraic solution:** Replacing b with 4 gives us $2\left(\frac{x-3}{5}\right)=4$. We multiply each side of the equation by $\frac{5}{2}$ to get $x-3=4\cdot\frac{5}{2}=10$. Finally, we add 3 to each side of this last equation to get $x=10+3=\mathbf{13}$.

Notes: (1) $2\left(\frac{x-3}{5}\right) = \frac{2}{5}(x-3)$. To get rid of the $\frac{2}{5}$, we multiply by its reciprocal (which is $\frac{5}{2}$).

(2) As an alternative to multiplying by $\frac{5}{2}$, we can first divide each side of the equation by 2, and then multiply each side of the resulting equation by 5. Here are the steps in detail:

$$2\left(\frac{x-3}{5}\right) = 4$$
$$\frac{2}{2}\left(\frac{x-3}{5}\right) = \frac{4}{2}$$
$$\frac{x-3}{5} = 2$$
$$5\left(\frac{x-3}{5}\right) = 2 \cdot 5$$
$$x - 3 = 10$$

(3) It's nice to practice solving these equations in your head informally. See if you can understand and then emulate the following reasoning.

What do we multiply 2 by to get 4? Well, 2 times 2 is 4. So $\frac{x-3}{5}$ must be 2. Now, what divided by 5 is 2. Well, 10 divided by 5 is 2. So $x-3$ must be 10. Finally, what number minus 3 is 10? Well, 13 minus 3 is 10. So x must be 13.

7. If $7x - 3 = 17$, what is the value of $14x - 5$?

*** Algebraic solution:** We add 3 to each side of the given equation to get $7x = 17 + 3$, or $7x = 20$. We now multiply each side of this last equation by 2 to get $14x = 40$. Finally, we subtract 5 from each side of this last equation to get $14x - 5 = 40 - 5 = \mathbf{35}$.

Notes: (1) We did *not* need to find x to solve this problem. Since the expression we are trying to find has $14x$ as one of its terms, it is more efficient to multiply $7x$ by 2, than it is to solve the original equation for x and then multiply by 14.

(2) If you didn't notice that you could change $7x$ into $14x$ by multiplying by 2, then the problem could be solved as follows:

$$7x - 3 = 17$$
$$7x = 20$$
$$x = \frac{20}{7}$$
$$14x = \frac{20}{7} \cdot 14$$
$$14x = 20 \cdot 2$$
$$14x = 40$$
$$14x - 5 = 40 - 5$$
$$14x - 5 = 35$$

8. Last month Josephine worked 7 less hours than Maria. If they worked a combined total of 137 hours, how many hours did Maria work that month?

*** Algebraic solution:** Let x be the number of hours Maria worked. Then Josephine worked $x - 7$ hours, and we have $x + (x - 7) = 137$. Therefore, $2x - 7 = 137$, and so $2x = 137 + 7 = 144$. So the number of hours that Maria worked is $x = \frac{144}{2} = \mathbf{72.}$

Note: This problem can also be solved by guessing. I leave the details of this solution to the reader.

LEVEL 1: GEOMETRY AND TRIG

9. What is the diameter of a circle whose area is 16π?

   (A) 4
   (B) 8
   (C) 16
   (D) 8π

Solution by starting with choice C: The area of a circle is $A = \pi r^2$. Let's start with choice C as our first guess. If $d = 16$, then $r = 8$, and so we have $A = \pi \cdot 8^2 = \pi \cdot 64 = 64\pi$. Since this is too big we can eliminate choices C and D.

Let's try choice B next. If $d = 8$, then $r = 4$, and so $A = \pi \cdot 4^2 = 16\pi$. This is correct, and so the answer is choice B.

Notes: (1) A **circle** is a two-dimensional geometric figure formed of a curved line surrounding a center point, every point of the line being an equal distance from the center point. This distance is called the **radius** of the circle. The **diameter** of a circle is the distance between any two points on the circle that pass through the center of the circle.

(2) The diameter of a circle is twice the radius of the circle.

$$d = 2r$$

*** Algebraic solution:** We use the area formula $A = \pi r^2$, and substitute 16π in for A.

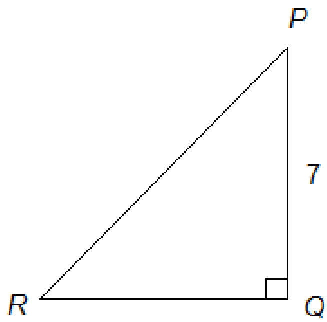
$$A = \pi r^2$$
$$16\pi = \pi r^2$$
$$16 = r^2$$
$$4 = r$$

Now, the diameter of a circle is twice the radius, and so we have $d = 2r = 2 \cdot 4 = 8$, choice B.

Note: The equation $r^2 = 16$ would normally have two solutions:

$r = 4$ and $r = -4$.

But the radius of a circle must be positive, and so we reject -4.



10. In the isosceles right triangle above, $PQ = 7$. What is the length, in inches, of $\overline{PR}$?

(A) $7\sqrt{2}$
(B) $\sqrt{14}$
(C) 14
(D) 7

Solution by the Pythagorean Theorem: Since the triangle is isosceles, $RQ = PQ = 7$. By the Pythagorean Theorem, we have

$$PR^2 = 7^2 + 7^2 = 49 + 49 = 49 \cdot 2.$$

So $PR = \sqrt{49 \cdot 2} = \sqrt{49} \cdot \sqrt{2} = 7\sqrt{2}$, choice A.

Remarks: (1) The Pythagorean Theorem says that if a right triangle has legs of length a and b, and a hypotenuse of length c, then $c^2 = a^2 + b^2$.

(2) The Pythagorean Theorem is one of the formulas given to you in the beginning of each math section.

(3) The equation $PR^2 = 49 \cdot 2$ would normally have two solutions:

$$PR = 7\sqrt{2} \text{ and } PR = -7\sqrt{2}.$$

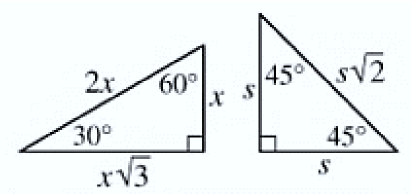
But the length of a side of a triangle cannot be negative, and so we reject $-7\sqrt{2}$.

(4) A **triangle** is a two-dimensional geometric figure with three sides and three angles. The sum of the degree measures of all three angles of a triangle is 180°.

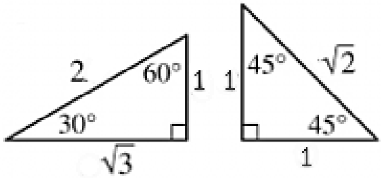
(5) A triangle is **isosceles** if it has two sides of equal length. Equivalently, an isosceles triangle has two angles of equal measure.

*** Solution using a 45, 45, 90 triangle:** An isosceles right triangle is the same as a 45, 45, 90 triangle, and so the hypotenuse has length $PR = 7\sqrt{2}$, choice A.

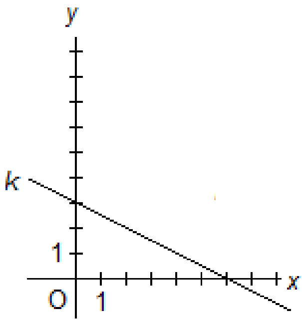
Note: The following two special triangles are given on the SAT:



Some students get a bit confused because there are variables in these pictures. We can simplify the pictures if we substitute a 1 in for the variables.



Notice that the sides of the 30, 60, 90 triangle are then 1, 2 and $\sqrt{3}$ and the sides of the 45, 45, 90 triangle are 1, 1 and $\sqrt{2}$. The variables in the first picture above just tell us that if we multiply one of the sides in the second picture by a number, then we have to multiply the other two sides by the same number. For example, instead of 1, 1 and $\sqrt{2}$, we can have 7, 7 and $7\sqrt{2}$ (here $s = 7$), or $\sqrt{2}$, $\sqrt{2}$, and 2 (here $s = \sqrt{2}$).



11. What is the equation of line k in the figure above?

(A) $2x + y = 3$

(B) $2x + y = 6$

(C) $x + 2y = 6$

(D) $x + 2y = 12$

Solution by plugging in points: Since the point $(0, 3)$ lies on the line, if we substitute 0 in for x and 3 in for y, we should get a true equation.

(A)	$3 = 3$	True
(B)	$3 = 6$	False
(C)	$6 = 6$	True
(D)	$6 = 12$	False

We can eliminate choices B and D because they came out False

The point $(6, 0)$ also lies on the line. So if we substitute 6 for x and 0 for y we should also get a true equation.

(A) $12 = 3$	False
(C) $6 = 6$	True

We can eliminate choice A because it came out False. Therefore, the answer is choice C.

*** Solution using the slope-intercept form of an equation of a line:** Recall that the slope-intercept form for the equation of a line is

$$y = mx + b.$$

$(0, 3)$ is the y-intercept of the point. Thus, b = 3. The slope of the given line is $m = \frac{\text{rise}}{\text{run}} = \frac{-3}{6}\ \ \frac{-1}{2}$. Thus, the equation of the line in slope-intercept form is $y = -\frac{1}{2}x + 3$.

We multiply each side of this equation by 2 to get $2y = -x + 6$. Finally, we add x to each side of this last equation to get $x + 2y = 6$, choice C.

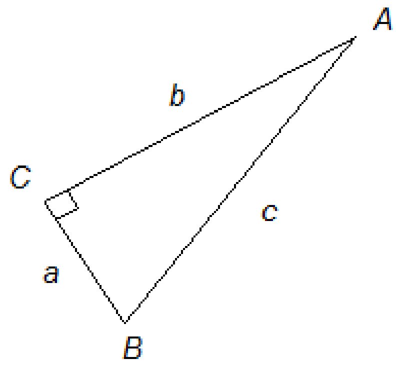
Notes: (1) To find the slope using the graph we simply note that to get from the y-intercept of the line to the x-intercept of the line we need to move down 3, then right 6.

(2) The answer choices are in **general form**. To change the equation of a line from slope-intercept form to general form we first eliminate all fractions by multiplying each side of the equation by the least common denominator. In this case, that is 2. Here are the steps in detail:

$$y = -\frac{1}{2}x + 3$$
$$2y = 2(-\frac{1}{2}x + 3)$$
$$2y = 2\left(-\frac{1}{2}x\right) + 2(3)$$
$$2y = -x + 6$$

We now simply add x to each side of this last equation to get

$$x + 2y = 6.$$



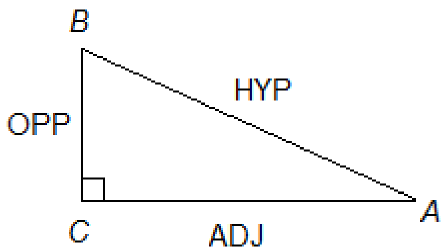
12. In the figure above, what is sin A ?

(A) $\frac{c}{b}$
(B) $\frac{a}{b}$
(C) $\frac{a}{c}$
(D) $\frac{b}{a}$

* $\sin A = \frac{\text{OPP}}{\text{HYP}} = \frac{a}{c}$, choice C.

Here is a quick lesson in **right triangle trigonometry** for those of you that have forgotten.

Let's begin by focusing on angle A in the following picture**:**



Note that the **hypotenuse** is ALWAYS the side opposite the right angle.

The other two sides of the right triangle, called the **legs**, depend on which angle is chosen. In this picture we chose to focus on angle A. Therefore, the opposite side is BC, and the adjacent side is AC.

Now you should simply memorize how to compute the six trig functions:

$$\sin A = \frac{\text{OPP}}{\text{HYP}} \qquad \csc A = \frac{\text{HYP}}{\text{OPP}}$$

$$\cos A = \frac{\text{ADJ}}{\text{HYP}} \qquad \sec A = \frac{\text{HYP}}{\text{ADJ}}$$

$$\tan A = \frac{\text{OPP}}{\text{ADJ}} \qquad \cot A = \frac{\text{ADJ}}{\text{OPP}}$$

Here are a couple of tips to help you remember these:

(1) Many students find it helpful to use the word SOHCAHTOA. You can think of the letters here as representing sin, opp, hyp, cos, adj, hyp, tan, opp, adj.

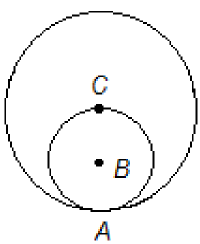
(2) The three trig functions on the right are the reciprocals of the three trig functions on the left. In other words, you get them by interchanging the numerator and denominator. It's pretty easy to remember that the reciprocal of tangent is cotangent. For the other two, just remember that the "s" goes with the "c" and the "c" goes with the "s." In other words, the reciprocal of sine is cosecant, and the reciprocal of cosine is secant.

To make sure you understand this, compute all six trig functions for each of the angles (except the right angle) in the triangle given in this problem. Please try this yourself before looking at the answers below.

$$\sin A = \frac{a}{c} \qquad \csc A = \frac{c}{a} \qquad \sin B = \frac{b}{c} \qquad \csc B = \frac{c}{b}$$

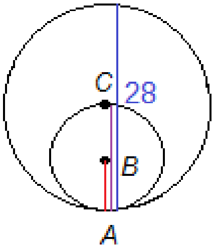
$$\cos A = \frac{b}{c} \qquad \sec A = \frac{c}{b} \qquad \cos B = \frac{a}{c} \qquad \sec B = \frac{c}{a}$$

$$\tan A = \frac{a}{b} \qquad \cot A = \frac{b}{a} \qquad \tan B = \frac{b}{a} \qquad \cot B = \frac{a}{b}$$



13. In the figure above, $A, B,$ and C lie on the same line. B is the center of the smaller circle, and C is the center of the larger circle. If the diameter of the larger circle is 28, what is the radius of the smaller circle?

Solution by assuming the figure is drawn to scale: We can assume that the figure is drawn to scale.

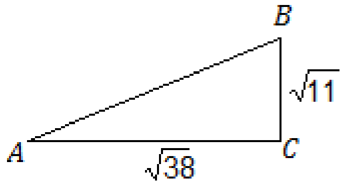


The three lines drawn in the picture above are just there for the purposes of measurement. In practice, you can just use your fingers to measure.

The smallest segment is the radius of the smaller circle. The longest segment is the diameter of the larger circle.

From the picture it is easy to see that the radius of the smaller circle is $\frac{1}{4}$ the diameter of the larger circle. So the answer is $\frac{1}{4} \cdot 28 = \mathbf{7}$.

*** Geometric solution:** Since the radius of a circle is half the diameter, the radius of the larger circle is $\frac{1}{2} \cdot 28 = 14$. This is also the diameter of the smaller circle. Therefore, the radius of the smaller circle is $\frac{1}{2} \cdot 14 = \mathbf{7}$.

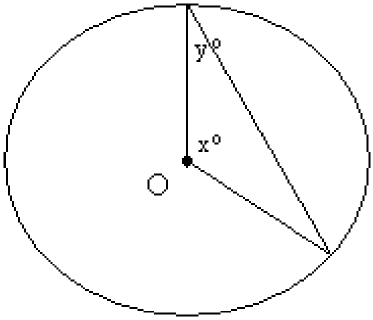


Note: Figure not drawn to scale.

14. In right triangle ABC above, what is the length of side AB?

*** Solution using the Pythagorean Theorem:** We use the Pythagorean Theorem: $c^2 = a^2 + b^2 = 38 + 11 = 49$. Therefore, $AB = c = \mathbf{7}$.

Note: See problem 10 for more information on the Pythagorean Theorem.



15. In the figure above, if $x = 122$ and O is the center of the circle, what is the value of y ?

*** Algebraic solution:** Note that the triangle is **isosceles**. In particular, y is equal to the measure of the unlabeled angle. Therefore, we have the following

$$x + y + y = 180$$
$$122 + 2y = 180$$
$$2y = 180 - 122$$
$$2y = 58$$
$$y = \frac{58}{2}$$
$$y = \mathbf{29}$$

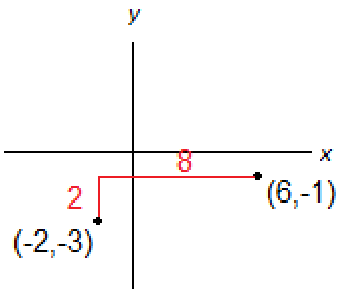
Notes: (1) See problem 10 for more information about isosceles triangles.

(2) All radii of a circle are equal. So if two sides of a triangle are radii of a circle, then the triangle must be isosceles.

(3) In an isosceles triangle, angles opposite the two equal sides have equal measure.

16. In the standard (x, y) coordinate plane, what is the slope of the line segment joining the points $(-2, -3)$ and $(6, -1)$?

Solution by drawing a picture: Let's plot the two points.



Note that to get from $(-2,-3)$ to $(6,-1)$ we move up 2 and right 8. Therefore, the answer is $2/8 = \mathbf{1/4}$ or $\mathbf{.25}$.

Note: If you cannot see where the 2 and 8 come from visually, then you can formally find the differences:

$$-1-(-3) = -1+3 = 2 \text{ and } 6-(-2) = 6+2 = 8.$$

*** Solution using the slope formula:** $\frac{-1-(-3)}{6-(-2)} = \frac{-1+3}{6+2} = \frac{2}{8} = \mathbf{1/4}$ or $\mathbf{.25}$.

Notes: (1) The slope of the line passing through the points (x_1, y_1) and (x_2, y_2) is

$$\text{Slope} = m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

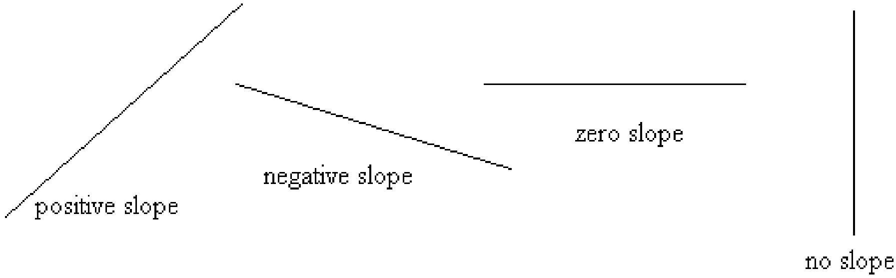
Here, the points are $(x_1, y_1) = (-2,-3)$ and $(x_2, y_2) = (6,-1)$.

(2) Lines with **positive slope** have graphs that go upwards from left to right.

Lines with **negative slope** have graphs that go downwards from left to right.

Horizontal lines have **zero slope**.

Vertical lines have **no slope** (or **infinite slope** or **undefined slope**).



LEVEL 1: PASSPORT TO ADVANCED MATH

$$3x(y + 2z)$$

17. Which of the following is equivalent to the expression above?

(A) $xy + 5xz$
(B) $3xy + 5xz$
(C) $3xy + 2z$
(D) $3xy + 6xz$

*** Solution using the distributive property:**

$$3x(y + 2z) = 3x \cdot y + 3x \cdot 2z = 3xy + 6xz$$

So the answer is choice D.

Notes: (1) The **distributive property** says that for all real numbers a, b, and c

$$a(b + c) = ab + ac$$

More specifically, this property says that the operation of multiplication distributes over addition. The distributive property is very important as it allows us to multiply and factor algebraic expressions.

In this problem, $a = 3x$, $b = y$, and $c = 2z$.

(2) $3x \cdot 2z = 3 \cdot 2 \cdot x \cdot z = 6xz$. Similarly, $3x \cdot y = 3xy$.

Solution by picking numbers: Let's choose values for x, y, and z, say $x = 2$, $y = 3$, and $z = 4$. Then

$$3x(y + 2z) = 3 \cdot 2(3 + 2 \cdot 4) = 6(3 + 8) = 6 \cdot 11 = \mathbf{66}.$$

Put a nice big, dark circle around this number so that you can find it easily later. We now substitute the numbers that we chose into each answer choice.

(A) $2 * 3 + 5 * 2 * 4 = 6 + 40 = 64$
(B) $3 * 2 * 3 + 5 * 2 * 4 = 18 + 40 = 58$
(C) $3 * 2 * 3 + 2 * 4 = 18 + 8 = 26$
(D) $3 * 2 * 3 + 6 * 2 * 4 = 18 + 48 = 66$

Since A, B and C are incorrect we can eliminate them. Therefore, the answer is choice D.

Notes: (1) D is **not** the correct answer simply because it is equal to 66. It is correct because all 3 of the other choices are **not** 66.

(2) See problem 1 for more information on picking numbers.

18. If $f(x) = 5(x-2) + 3$, which of the following is equivalent to $f(x)$?

(A) $3 - 10x$
(B) $5x - 7$
(C) $5x - 5$
(D) $5x + 1$

*** Solution using the distributive property:** We have

$$5(x-2) + 3 = 5x - 10 + 3 = 5x - 7$$

So $f(x) = 5x - 7$, choice B.

Notes: (1) The **distributive property** says that if a, b, and c are real numbers, then

$$a(b+c) = ab + ac.$$

In this question, $a = 5$, $b = x$, and $c = -2$.

So we have $5(x-2) = 5\big(x + (-2)\big) = 5x + 5(-2) = 5x - 10$.

(2) A common mistake would be to write $5(x-2) = 5x - 2$. This would lead to $5(x-2) + 3 = 5x - 2 + 3 = 5x + 1$. This is choice D which is **wrong!**

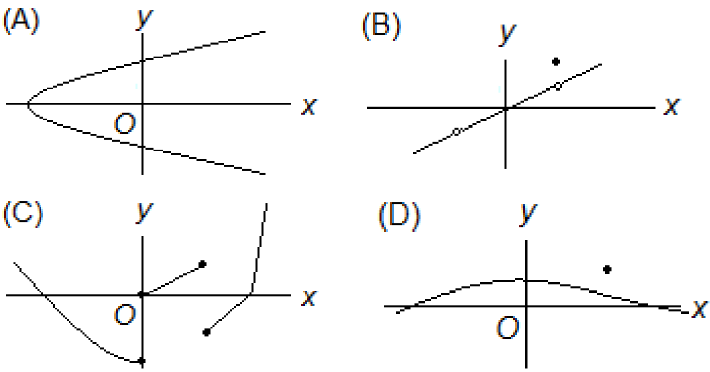
Solution by picking a number: Let's choose a value for x, say $x = 2$. It then follows that $f(x) = f(2) = 5(2-2)+3 = 5 \cdot 0 + 3 = \mathbf{3}$. Put a nice, big, dark circle around this number so that you can find it easily later. We now substitute $x = 2$ into each answer choice.

(A) $3 - 10x = 3 - 10 \cdot 2 = 3 - 20 = -17$
(B) $5x - 7 = 5 \cdot 2 - 7 = 10 - 7 = 3$
(C) $5x - 5 = 5 \cdot 2 - 5 = 10 - 5 = 5$
(D) $5x + 1 = 5 \cdot 2 + 1 = 10 + 1 = 11$

We now compare each of these numbers to the number that we put a nice big, dark circle around. Since A, C and D are incorrect we can eliminate them. Therefore, the answer is choice B.

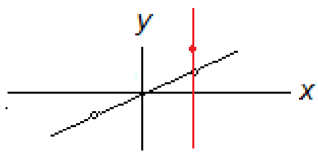
Important note: B is **not** the correct answer simply because it is equal to 3. It is correct because all three of the other choices are **not** 3. **You absolutely must check all four choices!**

19. Which of the following graphs is the graph of a function?



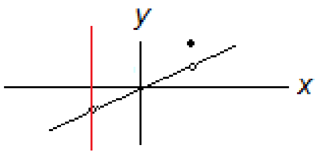
* **Direct solution:** Only choice B passes the **vertical line test**. In other words, any vertical line will hit the graph *at most* once. The answer is B.

Notes: (1) Observe how the following vertical line hits the graph *only* once:

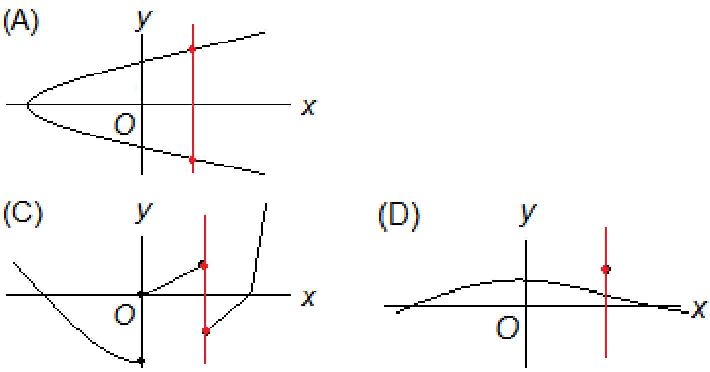


Only the solid dot is a point on the graph. The open circle indicates that there is no point there.

(2) There is also a vertical line that does not hit the graph at all. This is okay – a vertical line has to hit the graph *at most* once. This means one time *or* zero times.



Solution by process of elimination: To eliminate an answer choice, it suffices to draw a vertical line that hits the graph more than once.



This eliminates choices A, C, and D. So the answer is B.

20. If $3k^2 - 33 = 12 - 2k^2$, what are all possible values of k ?

    (A) 3 only
    (B) -3 only
    (C) 0 only
    (D) 3 and -3 only

Solution by plugging in the answer choices: According to the answer choices we need only check 0, 3, and -3.

$k = 0$: $3(0)^2 - 33 = 12 - 2(0)^2$ $-33 = 12$ False

$k = 3$: $3(3)^2 - 33 = 12 - 2(3)^2$ $-6 = -6$ True

$k = -3$: $3(-3)^2 - 33 = 12 - 2(-3)^2$ $-6 = -6$ True

So the answer is choice D.

Notes: (1) Since all powers of k in the given equation are even, 2 and −2 must give the same answer. So we didn't really need to check −2.

(2) Observe that when performing the computations above, the proper order of operations was followed. Exponentiation was done first, followed by multiplication, and then subtraction was done last.

For example, we have $3(3)^2 - 33 = 3 \cdot 9 - 33 = 27 - 33 = -6$ and $12 - 2(3)^2 = 12 - 2 \cdot 9 = 12 - 18 = -6$.

Order of Operations: Here is a quick review of order of operations.

PEMDAS	
P	Parentheses
E	Exponentiation
M	Multiplication
D	Division
A	Addition
S	Subtraction

Note that multiplication and division have the same priority, and addition and subtraction have the same priority.

*** Algebraic solution:** We add $2k^2$ to each side of the given equation to get $5k^2 - 33 = 12$. We then add 33 to get $5k^2 = 12 + 33 = 45$. Dividing each side of this last equation by 5 gives $k^2 = \frac{45}{5} = 9$. We now use the **square root property** to get $k = \pm 3$. So the answer is choice D.

Notes: (1) The equation $k^2 = 9$ has two solutions: $k = 3$ and $k = -3$. A common mistake is to forget about the negative solution.

(2) The **square root property** says that if $x^2 = c$, then $x = \pm\sqrt{c}$.

This is different from taking the positive square root of a number. For example, $\sqrt{9} = 3$, while the equation $x^2 = 9$ has two solutions $x = \pm 3$.

(3) Another way to solve the equation $k^2 = 9$ is to subtract 9 from each side of the equation, and then factor the difference of two squares as follows:

$$k^2 - 9 = 0$$
$$(k - 3)(k + 3) = 0$$

We now set each factor equal to 0 to get $k - 3 = 0$ or $k + 3 = 0$. Thus, $k = 3$ or $k = -3$.

21. A triangle has area A, base b, and height h. Which of the following represents b in terms of A and h ?

    (A) $b = \frac{A}{2h}$
    (B) $b = \frac{A}{h}$
    (C) $b = \frac{2A}{h}$
    (D) $b = \frac{\sqrt{A}}{h}$

*** Algebraic solution:** The area of a triangle is $A = \frac{1}{2}bh$. Multiplying each side of this equation by 2 gives $bh = 2A$. Dividing each side of this last equation by h gives $b = \frac{2A}{h}$, choice C.

Note: We can solve the equation $A = \frac{1}{2}bh$ for b in a single step by multiplying each side of the equation by $\frac{2}{h}$.

Solution by picking numbers: Let's let $b = 2$ and $h = 3$, so that $A = 3$. Put a nice big dark circle around **2** so you can find it easier later. We now substitute $A = 3$ and $h = 3$ into each answer choice:

(A) $w = \frac{3}{2\cdot 3} = \frac{1}{2} = 0.5$
(B) $w = \frac{3}{3} = 1$
(C) $w = \frac{2\cdot 3}{3} = 2$
(D) $w = \frac{\sqrt{3}}{3}$

Since A, B, and D each came out incorrect, the answer is choice C**.**

Important note: C is **not** the correct answer simply because it is equal to 3. It is correct because all three of the other choices are **not** 3. **You absolutely must check all four choices!**

Remark: All of the above computations can be done in a single step with your calculator (if a calculator is allowed for this problem).

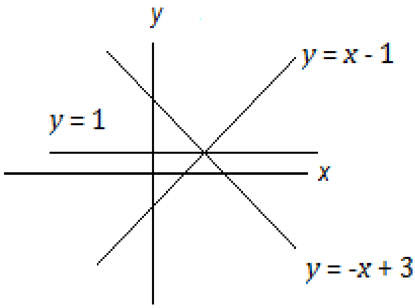
$$f(x) = 2x^3 - 5$$
$$g(x) = \frac{1}{2}x + 1$$

22. The functions f and g are defined above. What is the value of $g(4) - f(1)$?

* $g(4) = \frac{1}{2}(4) + 1 = 2 + 1 = 3.$

$f(1) = 2 \cdot 1^3 - 5 = 2 \cdot 1 - 5 = 2 - 5 = -3.$

Therefore, $g(4) - f(1) = 3 - (-3) = 3 + 3 = \mathbf{6}$.



23. Three equations and their graphs in the xy-plane are shown above. How many solutions does the system consisting of those three equations have?

* From the graphs we see that this system has one solution. It is the point of intersection of all 3 graphs. The answer is $\mathbf{1}$.

Notes: (1) The figure shows three graphs in the xy-plane. These are the graphs of the following system of equations:

$$\begin{aligned} y &= x - 1 \\ y &= -x + 3 \\ y &= \phantom{-x+{}}1 \end{aligned}$$

(2) To find the point of intersection of the three graphs, first observe that all three points must have y-coordinate 1 (because $y = 1$ is one of the equations). We can now substitute $y = 1$ into either of the other two equations to find x. For example, $1 = x - 1$ implies that $x = 2$. So the only solution to the given system is $(\mathbf{2}, \mathbf{1})$.

(3) Let's just check that the point $(2, 1)$ is also on the graph of the equation $y = -x + 3$. If we substitute 1 for x, we get $y = -1 + 3 = 2$.

24. If $y = kx$ and $y = 7$ when $x = 11$, then what is y when $x = 33$?

Algebraic solution: We are given that $y = 7$ when $x = 11$, so $7 = k(11)$, or $k = \frac{7}{11}$. Therefore $y = \frac{7x}{11}$. When $x = 33$, we have $y = \frac{7(33)}{11} = \mathbf{21}$.

Solution using direct variation: Since $y = kx$, y varies directly as x, and so $\frac{y}{x}$ is a constant. So we get the following ratio: $\frac{7}{11} = \frac{y}{33}$. Cross multiplying gives $7 \cdot 33 = 11y$, so that $y = \frac{7 \cdot 33}{11} = \mathbf{21}$.

Graphical solution: The graph *of* $y = f(x)$ is a line passing through the points $(0, 0)$ and $(11, 7)$ The slope of this line is $\frac{7-0}{11-0} = \frac{7}{11}$. Writing the equation of the line in slope-intercept form we have $y = \frac{7}{11}x$. As in solution 1, when $x = 33$, we have $y = \frac{7(33)}{11} = \mathbf{21}$.

*** Quick solution:** To get from $x = 11$ to $x = 33$ we multiply x by 3. So we have to also multiply y by 3. We get $3(7) = \mathbf{21}$.

Note: The following are all equivalent ways of saying the same thing:

(1) y varies directly as x.
(2) y is directly proportional to x.
(3) $y = kx$ for some constant k.
(4) $\frac{y}{x}$ is constant.
(5) The graph of $y = f(x)$ is a nonvertical line through the origin.

LEVEL 1: PROBLEM SOLVING AND DATA

25. A dentist sees 3 patients in 45 minutes. At this rate, how many patients would the dentist see in 3 hours?

(A) 9
(B) 12
(C) 14
(D) 15

Solution by setting up a ratio: We identify 2 key words. Let's choose "patients" and "minutes."

patients	3	x
minutes	45	180

Choose the words that are most helpful to you. Notice that we wrote in the number of patients next to the word patients, and the number of minutes next to the word minutes. Also notice that the number 45 is written under the number 3 because the dentist can see 3 patients in 45 minutes. Similarly, the number 180 is written under the unknown quantity x because we are trying to find out how many patients the dentist can see in 180 minutes ($= 3$ hours).

We now find x by cross multiplying and dividing.

$$\frac{3}{45} = \frac{x}{180}$$
$$45x = 3 \cdot 180$$
$$x = \frac{3 \cdot 180}{45} = 12$$

So the answer is choice B.

Notes: (1) At first glance it might seem to make more sense to choose "hours" as our second key word, but choosing the word "minutes" here will allow us to avoid having to work with fractions or decimals.

(2) There are 60 minutes in an hour. It follows that 3 hours is equal to $3 \cdot 60 = 180$ minutes.

(3) If we are not allowed to use a calculator for this problem, then a quick way to compute $\frac{3 \cdot 180}{45}$ is to first divide 180 by 45 to get 4. We can do this quickly in our head by counting how many 45's are in 180 (45, 90, 135, 180 – indeed there are 4). So we have

$$\frac{3 \cdot 180}{45} = 3 \cdot 4 = 12.$$

(4) If we choose to work in hours instead of minutes, then our initial setup would look as follows:

patients	3	x
hours	$.75$	3

Once again, we can find x by cross multiplying and dividing.

$$\frac{3}{.75} = \frac{x}{3}$$
$$.75x = 3 \cdot 3$$
$$x = \frac{3 \cdot 3}{.75} = 12$$

This gives us choice B.

*** Mental math:** 3 patients in 45 minutes is equivalent to 1 patient in 15 minutes. This is equivalent to 4 patients per hour. So in 3 hours the dentist will see $3 \cdot 4 = 12$ patients, choice B.

Questions 26 - 27 refer to the following information.

Favorite Animals

	Dog	Cat	Elephant	Monkey	Lion	Total
Fresh	82	17	20	36	18	173
Soph	51	46	5	50	6	158
Jun	24	30	63	22	30	169
Total	157	93	88	108	54	500

The table above lists the results of a survey of a random sample of 500 high school freshman, sophomores and juniors. Each student selected one animal that was his or her favorite.

26. How many sophomores selected the dog, cat, or lion as their favorite animal?

(A) 84
(B) 103
(C) 117
(D) 304

* We make sure to look at the row labeled “Soph” and we observe that 51 sophomores selected the dog, 46 sophomores selected the cat, and 6 sophomores selected the lion. So the number of sophomores that selected the dog, cat, or lion is $51 + 46 + 6 = 103$, choice B.

27. * What is the percentage of the 500 students that selected the elephant as their favorite animal?

(A) 1%
(B) 4%
(C) 12.6%
(D) 17.6%

* The total number of students is 500, and the number of these students that selected the elephant as their favorite animal is 88. So the desired percentage is $\frac{88}{500} \times 100 = 17.6\%$, choice D.

Notes: (1) To compute a percentage, use the simple formula

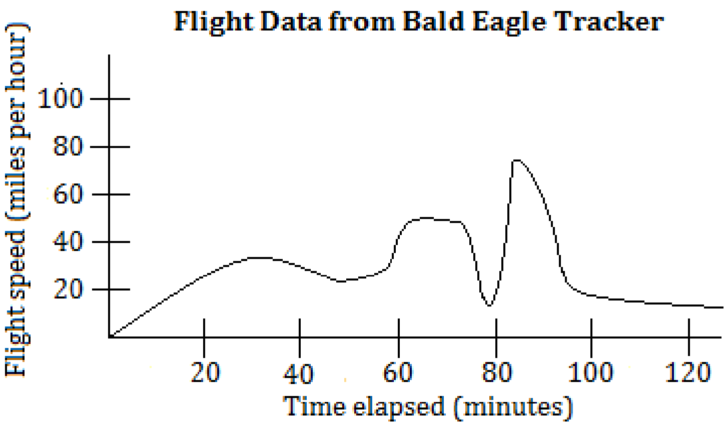
$$Percentage = \frac{Part}{Whole} \times 100$$

In this problem the *Part* is 88 and the *Whole* is 500.

(2) Alternatively we can simply divide the *Part* by the *Whole* and then change the resulting decimal to a percent by moving the decimal point to the right two places.

$$\frac{Part}{Whole} = \frac{88}{500} = 0.176 = 17.6\%$$

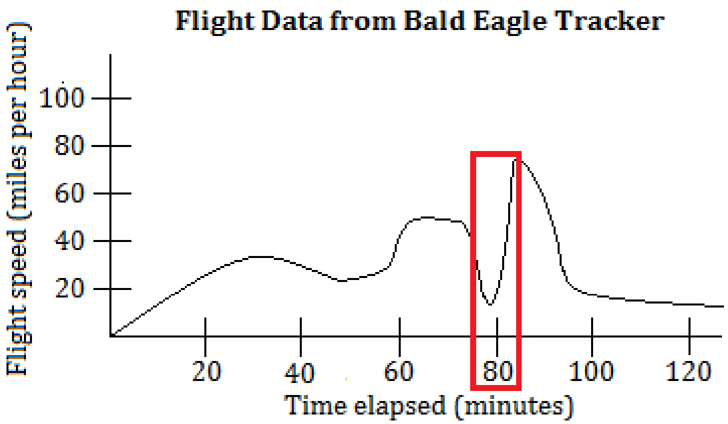
28. A tracker was implanted inside a bald eagle's wing, and its flight speed was monitored over a period of 2 hours. The data are graphed on the set of axes below with the time elapsed on the x-axis and the flight speed of the eagle on the y-axis. On which interval is the eagle's flight speed strictly decreasing then strictly increasing?



(A) Between 0 and 40 minutes
(B) Between 50 and 60 minutes
(C) Between 75 and 85 minutes
(D) Between 90 and 120 minutes

* We are looking for the graph to go down and then up as we move from left to right. This happens between 75 and 85 minutes, choice C.

Notes: (1) Let's isolate the part of the graph between 75 and 85 minutes.



Notice how in the boxed portion, the graph goes down, and then up, as we move from left to right.

(2) Between 0 and 40 minutes, the eagle's flight speed is strictly increasing then strictly decreasing.

(3) Between 50 and 60 minutes, the eagle's flight speed is strictly increasing.

(4) Between 90 and 120 minutes, the eagle's flight speed is strictly decreasing.

29. A data analyst was interested in the mean height of women in a small town. He randomly measured the heights of 200 women in that town, and found that the mean height of these women was 61 inches, and the margin of error for this estimate was 3 inches. The data analyst would like to repeat the procedure and attempt to reduce the margin of error. Which of the following samples would most likely result in a smaller margin of error for the estimated mean height of women in that same town?

    (A) 100 randomly selected people from the same town.
    (B) 100 randomly selected women from the same town.
    (C) 400 randomly selected people from the same town.
    (D) 400 randomly selected women from the same town.

* Increasing the sample size while keeping the population the same will most likely decrease the margin of error. So the answer is choice D.

Notes: (1) Decreasing the sample size will increase the margin of error. This allows us to eliminate choices A and B.

(2) The original sample consisted of only women. If we were to allow the second sample to include all people (including men), then we have changed the population. We cannot predict what impact this would have on the mean and margin of error. This allows us to eliminate choice C.

30. A lottery paid out a total of $6000 to players with winning tickets. Some winners received $500 and other winners received $1500. If at least one winner was paid $500 and at least one winner was paid $1500, what is one possible number of $1500 payouts?

* We subtract $500 and $1500 from the $6000 total to get $4000 remaining. Since 4000 is divisible by 500, it's possible that all of the remaining winners were paid $500. It follows that one possible number of $1500 payouts is **1**.

Notes: (1) The number of $1500 payouts can also be **2**, because if we subtract $2 \cdot 1500 = 3000$ from 6000, we get $6000 - 3000 = 3000$, and 3000 is divisible by 500.

Similarly, the number of $1500 payouts can be **3**, because if we subtract $3 \cdot 1500 = 4500$ from 6000, we get $6000 - 4500 = 1500$, and 1500 is divisible by 500.

(2) A complete list of the possible number of $1500 payouts is **1**, **2**, and **3**.

4 won't work because $4 \cdot 1500 = 6000$, and there would be no money left for the requirement of at least one $500 payout.

31. * A traffic sign on an expressway says that a driver must drive at least 45 miles per hour and at most 65 miles per hour. What is a possible amount of time, in hours, that it could take a driver to drive 585 miles, assuming that the driver obeys the traffic sign and does not make any stops?

* $585/65 = 9$ and $585 / 45 = 13$. So we can grid in any number between 9 and 13, inclusive.

Notes: (1) We can use the formula $d = rt$ (distance = rate × time). In this problem, it's easiest to use the formula in the form $t = \frac{d}{r}$.

(2) If the driver travels the whole 585 miles at 45 miles per hour, then the amount of time is $\frac{585}{45} = \mathbf{13}$ hours.

(3) If the driver travels the whole 585 miles at 65 miles per hour, then the amount of time is $\frac{585}{65} = \mathbf{9}$ hours.

(4) The driver may drive at a speed between 45 and 65 miles per hour, and he may change speed as he drives. This is why we can choose other values between 9 and 13 for the answer if we wish.

32. At a pet store 3 goldfish are selected at random from each group of 20. At this rate, how many goldfish will be selected in total if the pet store has 800 goldfish?

Solution by setting up a ratio: We identify 2 key words. Let's choose "selected" and "group."

selected	3	x
group	20	800

We now find x by cross multiplying and dividing.

$$\frac{3}{20} = \frac{x}{800}$$

$$20x = 3 \cdot 800$$

$$x = \frac{3 \cdot 800}{20} = 3 \cdot 40 = \mathbf{120}.$$

*** Quick solution:** $\frac{800}{20} \cdot 3 = 40 \cdot 3 = \mathbf{120}$.

Notes: (1) There are 800 goldfish in total, and there are 20 in each group. So dividing 800 by 20 gives us the number of groups.

$$\frac{800}{20} = 40 \text{ groups}$$

(2) Since 3 goldfish are selected from each group, and there are 40 groups, the total number of goldfish selected is $40 \cdot 3 = 120$.