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By Dr. Steve Warner

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Table of Contents

PROBLEMS BY LEVEL AND TOPIC WITH FULLY EXPLAINED SOLUTIONS

Note: The quickest solution will always be marked with an asterisk (*).

LEVEL 1: NUMBER THEORY

1. Which of the following numbers is a COUNTEREXAMPLE to the statement "Every positive integer greater than 23 is either prime or divisible by 3, 5, or 7" ?

   (A) 17
   (B) 21
   (C) 25
   (D) 91
   (E) 121

***** We are looking for a positive integer greater than 23 that is not prime AND not divisible by 3, 5, or 7.

25 is divisible by 5

91 is divisible by 7

$121 = 11^2$. So 121 is not prime and not divisible by 3, 5, or 7. So the answer is choice (E).

Note: We can eliminate 17 and 21 since they are *not* greater than 23.

Definitions: The **integers** are the counting numbers together with their negatives.

$$\{\ldots,-4, -3, -2, -1, 0, 1, 2, 3, 4,\ldots\}$$

The **positive integers** consist of the positive numbers from that set.

$$\{1, 2, 3, 4,\ldots\}$$

A **prime number** is a positive integer that has **exactly** two factors (1 and itself). Here is a list of the first few primes:

$$2, 3, 5, 7, 11, 13, 17, 19, 23,\ldots$$

Note that 1 is **not** prime. It only has one factor!

2. Marco is drawing a time line to represent a 500-year period of time. If he makes the time line 80 inches long and draws it to scale, how many inches will represent each year?

(A) 0.14
(B) 0.15
(C) 0.16
(D) 0.17
(E) 0.18

*** Solution by setting up a ratio:** This is a simple ratio. We begin by identifying 2 key words that tell us what 2 things are being compared. In this case, such a pair of key words is "years" and "inches."

years	500	1
inches	80	*x*

Now draw in the division symbols and equal sign, cross multiply and divide the corresponding ratio to find the unknown quantity *x*.

$$\frac{500}{80} = \frac{1}{x}$$

$$500x = (1)(80)$$

$$x = \frac{80}{500} = .16$$

This is choice (C).

3. When six given real numbers are multiplied together, the product is positive. Which of the following could be true about the six numbers?

(A) One is negative, four are positive, and one is zero.
(B) Two are negative, three are positive, and one is zero.
(C) One is negative and five are positive
(D) Three are negative and three are positive.
(E) Four are negative and two are positive.

* When we multiply several numbers together, the product will be positive if there is an even number of negative factors, and the rest are positive. The answer choice satisfying this condition is choice (E).

4. If the square root of the cube root of a number is 3, what is the number?

   (A) 729
   (B) 243
   (C) 187
   (D) 81
   (E) 27

Solution by starting with choice C: Let's start with choice C and guess that the number is 187. The cube root of 187 is approximately 5.7, and the square root of 5.7 is approximately 2.4. Since this is too small we can eliminate choices C, D, and E.

Let's try choice B next. The cube root of 243 is approximately 6.2, and the square root of 6.2 is approximately 2.5. So we can eliminate choice B, and the answer must be choice (A).

Let's just check that choice A is actually the answer. The cube root of 729 is 9, and the square root of 9 is 3. So the answer is (A).

Note: When plugging in or checking answer choices it is a good idea to start with choice C unless there is a specific reason not to. In this problem eliminating choice C allows us to eliminate two more answer choices.

Algebraic solution using radicals: If we let x represent the number, then we are given that $\sqrt{\sqrt[3]{x}} = 3$. Squaring each side of this equation gives $\sqrt[3]{x} = 3^2 = 9$. Cubing each side of this last equation gives

$$x = 9^3 = 729.$$

This is choice (A).

***Algebraic solution using fractional exponents:** If we let x represent the number, then we are given that $\left(x^{\frac{1}{3}}\right)^{\frac{1}{2}} = 3$ (See the tenth law in the table below). So we have $x^{\frac{1}{6}} = 3$ (See the fifth law in the table below). We now raise each side of this last equation to the sixth power to get

$$x = 3^6 = 729.$$

This is choice (A).

Laws of Exponents

Law	Example
$x^0 = 1$	$3^0 = 1$
$x^1 = x$	$9^1 = 9$
$x^a x^b = x^{a+b}$	$x^3 x^5 = x^8$
$x^a/x^b = x^{a-b}$	$x^{11}/x^4 = x^7$
$(x^a)^b = x^{ab}$	$(x^5)^3 = x^{15}$
$(xy)^a = x^a y^a$	$(xy)^4 = x^4 y^4$
$(x/y)^a = x^a/y^a$	$(x/y)^6 = x^6/y^6$
$x^{-1} = 1/x$	$3^{-1} = 1/3$
$x^{-a} = 1/x^a$	$9^{-2} = 1/81$
$x^{1/n} = \sqrt[n]{x}$	$x^{1/3} = \sqrt[3]{x}$
$x^{m/n} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$	$x^{9/2} = \sqrt{x^9} = \left(\sqrt{x}\right)^9$

Level 1: Algebra and Functions

5. If $y = 2x^3 - 3.2$, for what value of x is $y = 5$?

   (A) 1.6
   (B) 1.9
   (C) 2.4
   (D) 3.5
   (E) 4.1

Solution by starting with choice C: We start with choice C and take a guess that $x = 2.4$. We then have $y = 2(2.4)$ ^ 3 – 3.2 = 24.448 (just use your calculator). This is too big. So we can eliminate choices C, D, and E.

Let's try choice B next and guess that $x = 1.9$. Then

$$y = 2(1.9) \wedge 3 - 3.2 = 10.518.$$

This is still too big. So we can eliminate choice B and the answer must be choice (A).

Let's just verify that choice A does actually give the correct answer. If $x = 1.6$, then $y = 2(1.6)$ ^ 3 – 3.2 = 4.992. This is close enough. So the answer is in fact choice (A).

*** Algebraic solution:** Let's substitute 5 in for y to get $5 = 2x^3 - 3.2$.

We now add 3.2 to each side of this equation to get $8.2 = 2x^3$.

Now divide each side of this equation by 2 to get $4.1 = x^3$

Finally, take the cube root of each side of this equation (use your calculator) to get $x \approx 1.6$, choice (A).

6. If $3a + ab = 56$ and $b - 8 = -4$, what is the value of a ?

(A) 2
(B) 4
(C) 6
(D) 8
(E) 10

*** Algebraic solution:** We start by solving the second equation for b to get $b = 4$. We then substitute $b = 4$ into the first equation to get

$$3a + 4a = 56.$$

So we have $7a = 56$, and therefore $a = 8$, choice (D).

Notes: (1) We can solve the equation $b - 8 = -4$ informally by asking "what number minus 8 gives us -4?" Well, this number is 4.

(2) If you prefer to solve the equation $b - 8 = -4$ formally, simply add 8 to each side of the equation.

(3) Similarly, we can solve the equation $7a = 56$ informally by asking "7 times what number is 56?" Well, this number is 8.

(4) If you prefer to solve the equation $7a = 56$ formally, simply divide each side of the equation by 7.

7. If $f(x) = \frac{3}{x^2}$ for $x > 0$, then $f(2.5) =$

(A) $\frac{8}{25}$

(B) $\frac{2}{5}$

(C) $\frac{12}{25}$

(D) $\frac{3}{5}$

(E) $\frac{18}{25}$

* $f(2.5) = \frac{3}{2.5^2} = \frac{12}{25}$, choice (C).

Notes: (1) We can do this computation right in our calculator:

3 / 2.5 ^ 2 ENTER

The output will be .48

(2) We can change .48 to a fraction in our TI-84 calculator by pressing MATH ENTER ENTER.

Alternatively, we can change each of the fractions in the answer choices to decimals by performing the appropriate division. For example, to change $\frac{12}{25}$ to a decimal we simply divide 12 by 25 in our calculator.

8. If $3x - 7x = 2x - 8x + 6$, then x =

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

*** Algebraic solution:** We start by combing like terms on each side of the equation to get $-4x = -6x + 6$. We now add $6x$ to each side of this last equation to get $2x = 6$. Finally, we divide by 2 to get $x = 3$, choice (D).

Remark: We can also solve this problem by starting with choice C. I leave the details of this solution to the reader.

9. If $2x^2 - 11 = 5 - 2x^2$, what are all possible values of x ?

(A) 2 only
(B) -2 only
(C) 0 only
(D) 2 and -2 only
(E) 0, 2, and -2

Solution by plugging in the answer choices: According to the answer choices we need only check 0, 2, and -2.

$x = 0$: $2(0)^2 - 11 = 5 - 2(0)^2$ $-11 = 5$ False

$x = 2$: $2(2)^2 - 11 = 5 - 2(2)^2$ $-3 = -3$ True

$x = -2$: $2(-2)^2 - 11 = 5 - 2(-2)^2$ $-3 = -3$ True

So the answer is choice (D).

Note: Since all powers of x in the given equation are even, 2 and -2 must give the same answer. So we didn't really need to check -2.

*** Algebraic solution:** We add $2x^2$ to each side of the given equation to get $4x^2 - 11 = 5$. We then add 11 to get $4x^2 = 16$. Dividing each side of this last equation by 4 gives $x^2 = 4$. We now use the **square root property** to get $x = \pm 2$. So the answer is choice (D).

Notes: (1) The equation $x^2 = 4$ has two solutions: $x = 2$ and $x = -2$. A common mistake is to forget about the negative solution.

(2) The **square root property** says that if $x^2 = c$, then $x = \pm\sqrt{c}$.

This is different from taking the positive square root of a number. For example, $\sqrt{4} = 2$, whereas the equation $x^2 = 4$ has two solutions $x = \pm 2$.

(3) Another way to solve the equation $x^2 = 4$ is to subtract 4 from each side of the equation, and then factor the difference of two squares as follows:

$$x^2 - 4 = 0$$
$$(x-2)(x+2) = 0$$

We now set each factor equal to 0 to get $x - 2 = 0$ or $x + 2 = 0$.

So $x = 2$ or $x = -2$.

10. If $x^2 - y^2 = 36$ and $x - y = -9$, then $x + y =$

(A) -9
(B) -4
(C) 0
(D) 4
(E) 9

* Substituting into the formula $(x+y)(x-y) = x^2 - y^2$ we have

$$(x+y)(-9) = 36.$$

It follows that $x + y = \frac{36}{-9} = -4$, choice (B).

11. If $a^{5c-2} = a^{3c+7}$ for all real values of a, what is the value of c ?

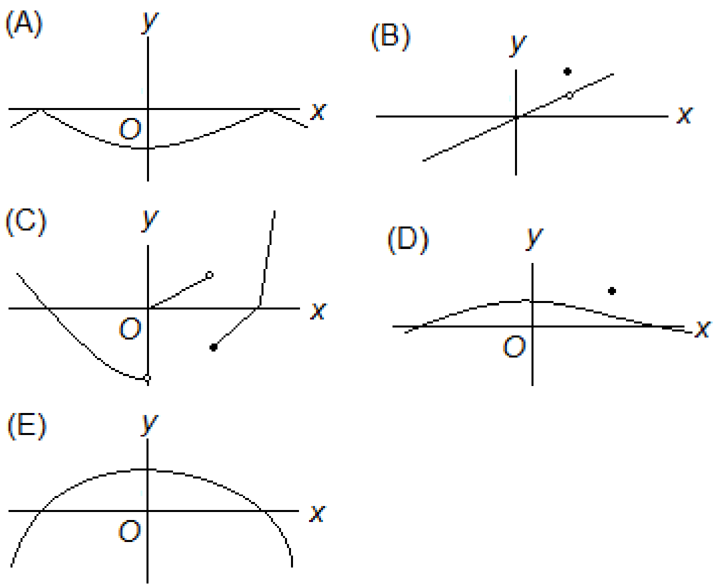
(A) 2.5
(B) 3
(C) 3.5
(D) 4
(E) 4.5

* **Algebraic solution:** Since the bases are the same we set the exponents equal to each other. So we have $5c - 2 = 3c + 7$. We subtract $3c$ from each side of this equation to get $2c - 2 = 7$. We now add 2 to each side of this last equation to get $2c = 9$. Finally, $c = \frac{9}{2} = 4.5$, choice (E).

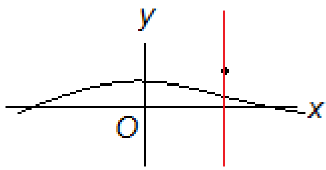
Notes: (1) Here we used the fact that if $a^x = a^y$, then $x = y$.

(2) This problem can also be solved by plugging in the answer choices. I leave the details to the reader.

12. Which of the following graphs could not be the graph of a function?



* Only choice (D) fails the **vertical line test**. In other words, we can draw a vertical line that hits the graph more than once:



So the answer is choice (D).

13. If $7x + y = 6$ and $5x + y = 2$, what is the value of $6x + y$?

   (A) -8
   (B) 4
   (C) 6
   (D) 12
   (E) 18

*** Solution by adding the two equations:** We add the two equations:

$$\begin{array}{r} 7x + \ \ y = 6 \\ \underline{5x + \ \ y = 2} \\ 12x + 2y = 8 \end{array}$$

Now observe that $12x + 2y = 2(6x + y)$. So $6x + y = \frac{8}{2} = 4$, choice (B).

Solution by subtracting the equations: We subtract the two equations to isolate x.

$$\begin{array}{r} 7x + \ \ y = 6 \\ \underline{5x + \ \ y = 2} \\ 2x \qquad = 4 \end{array}$$

So $x = 2$. Substituting $x = 2$ back into the first equation we get

$$\begin{array}{r} 7(2) + y = 6 \\ 14 \ + y = 6 \\ y = -8 \end{array}$$

So, $6x + y = 6(2) - 8 = 12 - 8 = 4$, choice (B).

14. If $|4 - 7x| > 30$, which of the following is a possible value of x?

   (A) -4
   (B) -2
   (C) 2
   (D) 3
   (E) 4

Solution by starting with choice C: Let's start with choice (C) and guess that $x = 2$. Then $|4 – 7x| = |4 – 7 \cdot 2| = |4 – 14| = |-10| = 10$. Since 10 is not greater than 30 we can eliminate choice (C). A moment's thought may lead you to suspect choice (A) (if you do not see this it is okay – just keep trying the answer choices until you get to it). Now, setting $x = -4$ gives us $|4 – 7x| = |4 – 7(-4)| = |4 + 28| = |32| = 32$. Since 32 is greater than 30, the answer is choice (A).

*** Partial algebraic solution:** We can try to simply eliminate the absolute values and solve the resulting inequality.

$$4 - 7x > 30$$
$$-7x > 26$$
$$x < \frac{26}{-7} \sim -3.714$$

Since $-4 < -3.714$, the answer is choice (A).

Note: The inequality changed direction in the last step because we divided each side of the inequality by a negative number.

Complete algebraic solution: The given absolute value inequality is equivalent to $4 - 7x < -30$ or $4 - 7x > 30$. Let's solve these two inequalities.

$$4 - 7x < -30 \quad \text{or} \quad 4 - 7x > 30$$
$$-7x < -34 \quad \text{or} \quad -7x > 26$$
$$x > \frac{34}{7} \sim 4.857 \quad \text{or} \quad x < \frac{26}{-7} \sim -3.714$$

So $x < -3.714$ or $x > 4.857$. Since $-4 < -3.714$, the answer is choice (A).

LEVEL 1: GEOMETRY

15. If the point $(3, y)$ is the intersection of the graphs of $x^2 = 9$ and $x = \frac{8y^3}{9}$, then $y =$

    (A) -3

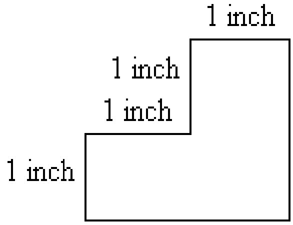
    (B) $-\frac{3}{2}$

    (C) 1

    (D) $\frac{3}{2}$

    (E) 2

* A point of intersection of two graphs lies on both graphs. In particular, the point $(3, y)$ lies on the graph of $x = \frac{8y^3}{9}$, and it follows that $3 = \frac{8y^3}{9}$. We multiply this equation by $\frac{9}{8}$ to get $y^3 = \frac{27}{8}$. So $y = \frac{3}{2}$, choice (D).



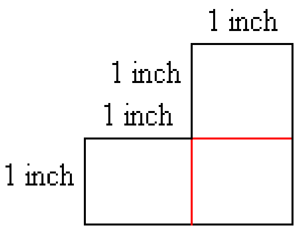
16. How many figures of the size and shape above are needed to completely cover a rectangle measuring 60 inches by 40 inches?

    (A) 200
    (B) 400
    (C) 600
    (D) 800
    (E) 1000

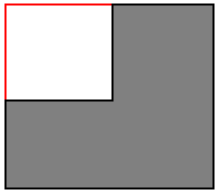
* **Solution by dividing areas:** The area of the given figure is 3 square inches and the area of the rectangle is $60 \cdot 40 = 2400$ square inches. We can see how many of the given figures cover the rectangle by dividing the two areas.

$$\frac{2400}{3} = 800\text{, choice (D).}$$

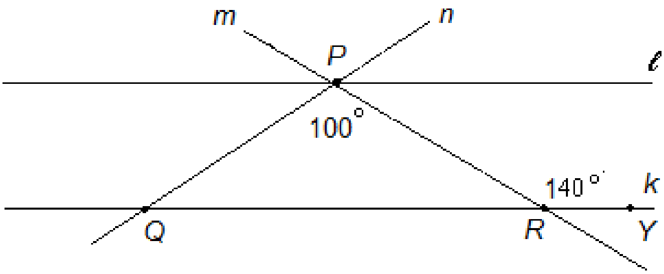
Note: We can get the area of the given figure by splitting it into 3 squares each with area 1 inch^2 as shown below. Then 1 + 1 + 1 = 3.



Another way to get the area of the given figure is to think of it as lying inside a square of side length 2 inches as shown below.



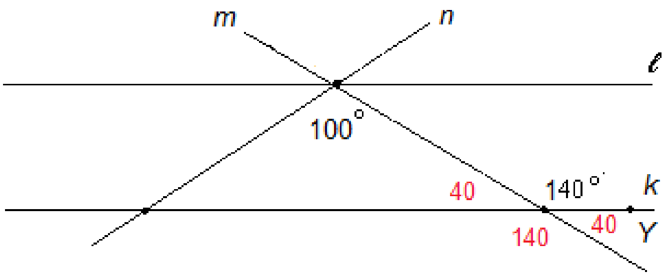
The area of the big square is 2 · 2 = 4 square inches, and the area of the small square is 1 · 1 = 1 square inch. Therefore the area of the given figure is 4 – 1 = 3 square inches.



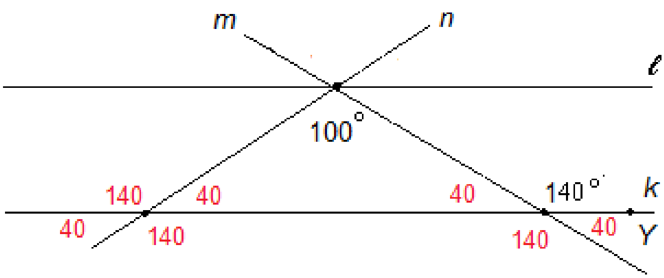
17. In the figure above, line ℓ is parallel to line k. Transversals and n intersect at point P on ℓ and intersect k at points R and Q, respectively. Point Y is on k, the measure of $\angle PRY$ is 140°, and the measure of $\angle QPR$ is 100°. How many of the angles formed by rays ℓ, k, m, and n have measure 40° ?

(A) 4
(B) 6
(C) 8
(D) 10
(E) 12

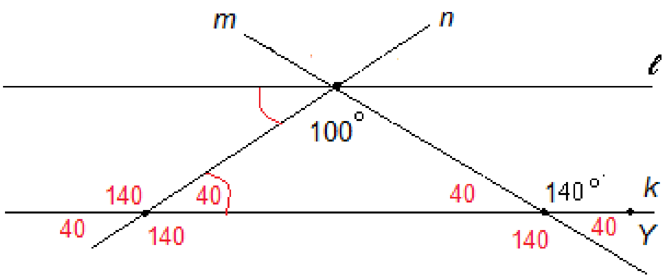
* is supplementary with $\angle PRY$. So $m\angle QRP$ is $180 - 140 = 40°$. We can then use vertical angles to get the remaining angles in the lower right hand corner.



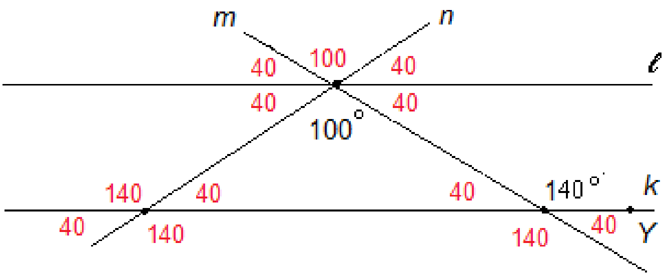
We now use the fact that the sum of the angle measures in a triangle is 180° to get that the measure of the third angle of the triangle is $180 - 100 - 40 = 40°$. We then once again use supplementary and vertical angles to get the remaining angles in the lower left hand corner.



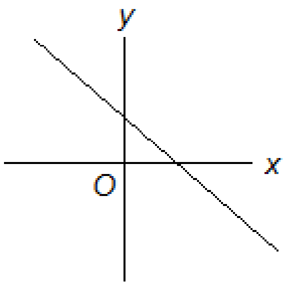
Now notice the following alternate interior angles.



Since alternate interior angles are congruent, we see that the angle marked above has a measure of 40°. We use supplementary and vertical angles to find the remaining angle measures.



Finally, we see that there are eight angles with measure 40°, choice (C).



18. The figure above shows the graph of the linear function $f(x) = mx + b$. Which of the following is true about m and b ?

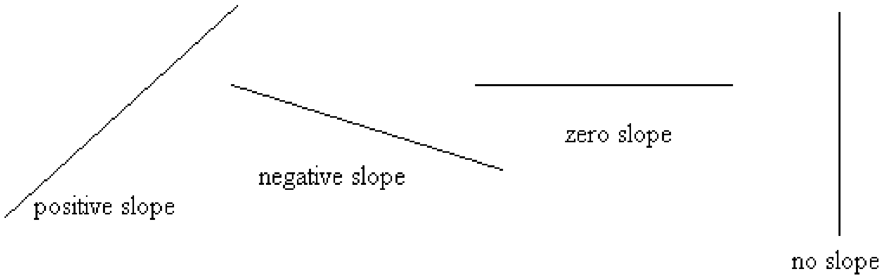
(A) $m > 0$ and $b > 0$
(B) $m > 0$ and $b < 0$
(C) $m < 0$ and $b > 0$
(D) $m < 0$ and $b < 0$
(E) $m = 0$ and $b > 0$

* Since the graph is going downwards from left to right, $m < 0$. Since the graph hits the y-axis above the x-axis, $b > 0$. So the answer is (C).

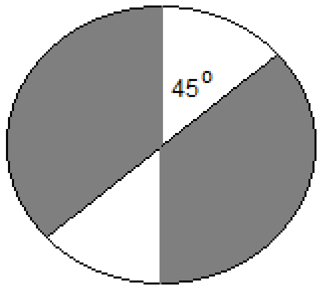
Slope formula and linear equations:

$$\text{Slope} = m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

Note: Lines with positive slope have graphs that go upwards from left to right. Lines with negative slope have graphs that go downwards from left to right. If the slope of a line is zero, it is horizontal. Vertical lines have **no** slope, or **undefined** slope (this is different from zero slope).



The **slope-intercept form of an equation of a line** is $\boldsymbol{y = mx + b}$ where m is the slope of the line and b is the y-coordinate of the y-intercept, i.e. the point $(0, b)$ is on the line. Note that this point lies on the y-axis.

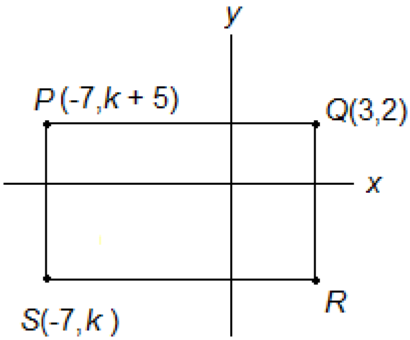


19. In the figure above, what percentage of the circle is shaded?

    (A) 65%
    (B) 70%
    (C) 75%
    (D) 80%
    (E) 85%

*** Solution by setting up a ratio:** From the figure, the unshaded portion of the circle represents 2(45) = 90° of the circle. Since there are 360° in a circle, the shaded portion represents 360 – 90 = 270° of the circle. So we have $\frac{270}{360} = 0.75 = 75\%$ of the circle is shaded, choice (C).

Note: To change a decimal to a percent, multiply by 100, or equivalently move the decimal point two places to the right.



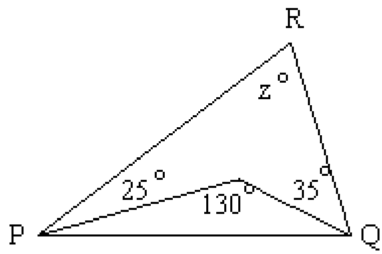
20. In rectangle $PQRS$ above, what are the coordinates of vertex R.

(A) (2, –3)
(B) (3, –2)
(C) (3, –2)
(D) (3, –3)
(E) (3, –4)

* First note that P and Q have the same y-coordinate so that $k + 5 = 2$. It follows that $k = 2 - 5 = -3$.

Now note that R has the same y-coordinate as S, so that the y-coordinate of R is $k = -3$.

Finally note that R has the same x-coordinate as Q, so the x-coordinate of R is 3. Therefore the answer is choice (D).

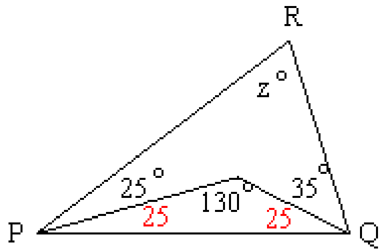


Note: Figure not drawn to scale.

21. In triangle PQR above, what is the value of z ?

(A) 65
(B) 70
(C) 75
(D) 80
(E) 85

Solution by picking numbers: Since every triangle has 180 degrees we choose values for the angle measures of the small triangle that add up to 180 – 130 = 50, say 25 and 25.



Note: Figure not drawn to scale.

We now see that z = 180 – 25 – 25 – 35 – 25 = 70, choice (B).

Remark: We could have chosen **any** two numbers that add up to 50 for the angles of the small triangle.

*** Geometric solution:** The two unlabeled angles in the smaller triangle must add up to 50. Therefore

$$z = 180 - 25 - 35 - 50 = 70, \text{ choice (B).}$$

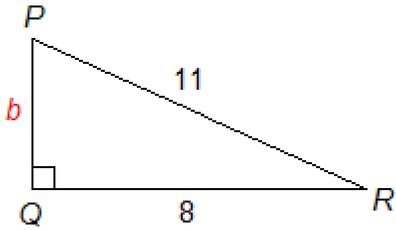
22. Right triangle PQR has a hypotenuse of length 11 and one of its legs as length 8. How many possible values are there for the area of triangle PQR ?

    (A) one
    (B) two
    (C) three
    (D) infinitely many
    (E) no such triangle exists

*** Geometric solution:** By the Pythagorean Theorem, the second leg of the right triangle is uniquely determined. The base and height of a right triangle are the two legs. Therefore the area of the triangle is also uniquely determined, and so the answer is choice (A).

Note: There is no need to actually compute the area of the triangle to solve this problem.

Solution by drawing a picture: Let's draw a picture:



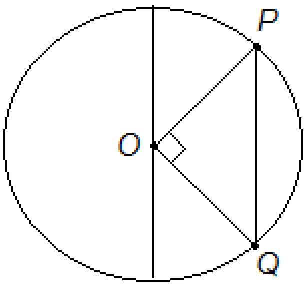
We can now find b by using the Pythagorean Theorem.

$$8^2 + b^2 = 11^2$$
$$64 + b^2 = 121$$
$$b^2 = 57$$
$$b = \sqrt{57}$$

So the area of the triangle is $A = \frac{1}{2} \cdot 8\sqrt{57} = 4\sqrt{57}$.

In particular, there is exactly one possible value for the area of the triangle, choice (A).

Note: The equation $b^2 = 57$ would normally have two solutions: $b = \sqrt{57}$ and $b = -\sqrt{57}$. But the length of a side of a triangle cannot be negative, so we reject $-\sqrt{57}$.



23. The circle shown above has center O and radius 7. What is the length of chord PQ ?

(A) $\frac{7}{2}$

(B) $\frac{7\sqrt{2}}{2}$

(C) 7

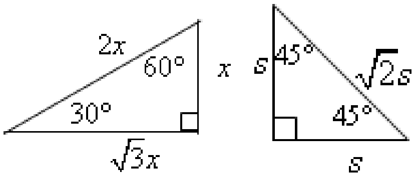
(D) $7\sqrt{2}$

(E) $7\sqrt{3}$

*** Solution using a 45, 45, 90 triangle:** Since the two legs of the right triangle are both radii of the circle, the triangle is an isosceles right triangle, or equivalently a 45, 45, 90 right triangle. It follows that PQ has length $7\sqrt{2}$, choice (D).

Notes: (1) A triangle is **isosceles** if it has two sides of equal length. Equivalently, an isosceles triangle has two angles of equal measure.

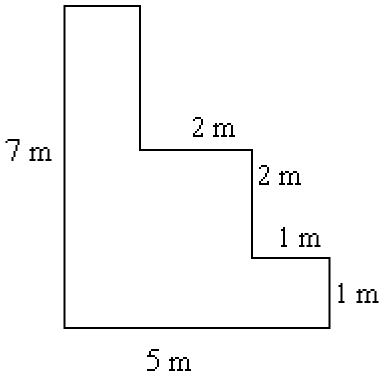
(2) For the SAT Math Subject Test, it is worth knowing the following two special triangles:



Some students get a bit confused because there are variables in these pictures. But the pictures become simplified if we substitute a 1 in for the variables. Then the sides of the 30, 60, 90 triangle are 1, 2 and $\sqrt{3}$ and the sides of the 45, 45, 90 triangle are 1, 1 and $\sqrt{2}$. The variable just tells us that if we multiply one of these sides by a number, then we have to multiply the other two sides by the same number.

For example, instead of 1, 1 and $\sqrt{2}$, we can have 3, 3 and $3\sqrt{2}$ (here we have $s = 3$), or $\sqrt{2}$, $\sqrt{2}$, and 2 (here $s = \sqrt{2}$). For this problem, we are using the 30, 60, 90 right triangle and $x = 7$.

(3) The hypotenuse of the right triangle can also be found using the Pythagorean Theorem. I leave the details of this solution to the reader.

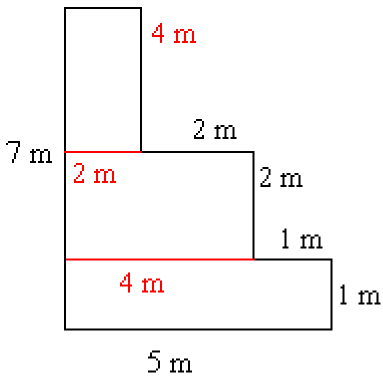


Note: Figure not drawn to scale.

24. What is the area of the figure above?

(A) 15 m^2
(B) 17 m^2
(C) 19 m^2
(D) 21 m^2
(E) 23 m^2

* We break the figure up into 3 rectangles and compute the length and width of each rectangle.



Note: Figure not drawn to scale.

The length and width of the bottom rectangle are 5 and 1 making the area $5 \cdot 1 = 5$ m^2.

The length of the middle rectangle is $5 - 1 = 4$, and the width is given as 2. Thus, the area is $4 \cdot 2 = 8$ m^2.

The length of the top rectangle is $4 - 2 = 2$, and the width is $7 - 1 - 2 = 4$. Thus, the area is $2 \cdot 4 = 8$ m^2.

We then get the total area by adding up the areas of the three rectangles: $5 + 8 + 8 = 21$ m^2, choice (D).

Remark: Notice that if we have the full length of a line segment, and one partial length of the same line segment, then we get the other length by subtracting the two given lengths.

LEVEL 1: PROBABILITY AND STATISTICS

25. Of the marbles in a jar, 14 are orange. Sarah randomly takes one marble out of the jar. If the probability is $\frac{2}{7}$ that the marble she chooses is orange, how many marbles are in the jar?

    (A) 4
    (B) 14
    (C) 28
    (D) 49
    (E) 98

Solution by plugging in answer choices: Let's start with choice C and guess that there are 28 marbles in the jar. We have $\frac{2}{7} \cdot 28 = 8$. This is too small. So we can eliminate choices A, B, and C.

Let's try choice D next and guess that there are 49 marbles in the jar. We have $\frac{2}{7} \cdot 49 = 14$. This is correct. So the answer is choice (D).

Remarks: (1) Saying that the probability is $\frac{2}{7}$ that an orange marble will be chosen is equivalent to saying that $\frac{2}{7}$ of the marbles are orange. So for example, when we guess that there are 49 marbles in the jar, we need to compute $\frac{2}{7}$ of 49. The word "of" always means multiplication.

(2) All of the computations above can be done by hand or in your calculator. For example, to compute $\frac{2}{7} \cdot 28$, simply type 2 / 7 * 28 followed by ENTER. The output will be 8.

*** Algebraic solution:** Let x be the total number of marbles in the jar. We are given that $\frac{2}{7}x = 14$. We multiply each side of this equation by $\frac{7}{2}$ to get that $x = 14 \cdot \frac{7}{2} = 49$, choice (D).

26. Between Town A and Town B there are 5 roads, between Town B and Town C there are 2 roads, and between Town C and Town D there are 4 roads. If a traveler were to travel from Town A to Town D, passing first through B, then through C, how many different routes does he have to choose from?

    (A) 11
    (B) 20
    (C) 40
    (D) 60
    (E) 80

*** Solution using the counting principle:** The counting principle says that when you perform events in succession you multiply the number of possibilities. So we get $5 \cdot 2 \cdot 4 = 40$ different routes, choice (C).

Remark: The **counting principle** says that if one event is followed by a second independent event, the number of possibilities is multiplied.

More generally, if $E_1, E_2, \dots, E_n$ are n independent events with $m_1, m_2, \dots, m_n$ possibilities, respectively, then event E_1 followed by event E_2, followed by event E_3, ..., followed by event E_n has $m_1 \cdot m_2 \cdots m_n$ possibilities.

In this question there are 3 events: "choosing a road between Town A and Town B," "choosing a road between Town B and Town C," and "choosing a road between Town C and Town D."

27. Kenneth's test average after 7 tests was 81. His score on the 8th test was 92. If all 8 tests were equally weighted, which of the following is closest to his test average after 8 tests?

    (A) 80
    (B) 82
    (C) 84
    (D) 86
    (E) 88

*** Solution by changing the average to a sum:** We change the average to a sum using the formula

$$\text{Sum} = \text{Average} \cdot \text{Number}$$

We are averaging seven numbers. Thus, the **Number** is 7. The **Average** is given to be 81. So the **Sum** of the seven numbers is $81 \cdot 7 = 567$.

When we add 92, the **Sum** becomes $567 + 92 = 659$. The **Average** is then —— $= 82.375$. This is closest to 82, choice (B).

28. Nine different books are to be stacked in a pile. One book is chosen for the bottom of the pile and another book is chosen for the top of the pile. In how many different orders can the remaining books be placed on the stack?

(A) 36
(B) 72
(C) 5040
(D) 40,320
(E) 362,880

* Once we have chosen a book for the bottom and a book for the top of the pile there are seven books left to stack. Thus, it follows that there are $7! = (7)(6)(5)(4)(3)(2)(1) = 5040$ ways to stack these books, choice (C).