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PROBLEMS BY LEVEL AND TOPIC WITH FULLY EXPLAINED SOLUTIONS

Note: The quickest solution will always be marked with an asterisk (*).

LEVEL 1: NUMBER THEORY

1. Which of the following sequences of inequalities expresses a true relationship between 1, $\frac{\pi}{2}$, and $\frac{e}{3}$?

   (A) $1 < \frac{\pi}{2} < \frac{e}{3}$
   (B) $1 < \frac{e}{3} < \frac{\pi}{2}$
   (C) $\frac{\pi}{2} < 1 < \frac{e}{3}$
   (D) $\frac{\pi}{2} < \frac{e}{3} < 1$
   (E) $\frac{e}{3} < 1 < \frac{\pi}{2}$

Solution by changing to decimals: We divide in our calculator to get

$$\frac{\pi}{2} = \pi / 2 \approx 1.57, \text{ and } \frac{e}{3} = e / 3 \approx .906.$$

Since .906 < 1 < 1.57, we have $\frac{e}{3} < 1 < \frac{\pi}{2}$, choice (E).

*** Quick mental solution:** Since $\pi \approx 3.14$, it is clear that $\frac{\pi}{2} > 1$. Since $e \approx 2.71$, it is clear that $\frac{e}{3} < 1$. So $\frac{e}{3} < 1 < \frac{\pi}{2}$, choice (E).

2. $\frac{7!}{4!-3!} =$

   (A) $5! + 4!$
   (B) $2(5! + 4!)$
   (C) $5! + 4! - 4$
   (D) $2(5! + 4! - 4)$
   (E) $2(5! + 4! - 3! + 2! - 1!)$

***** We first use our calculator to compute $\frac{7!}{4!-3!}$ = 7! / (4! – 3!) = **280**.

We then start with choice (C) and compute

$$5! + 4! - 4 = 140.$$

Since this is half of the correct answer, the answer is choice (D).

Note: When plugging in or checking answer choices it is a good idea to start with choice (C) unless there is a specific reason not to. In this particular question it does not matter, but in some questions eliminating choice (C) allows us to eliminate two other answer choices as well.

Definition: The **factorial** of a positive integer n, written $n!$, is the product of all positive integers less than or equal to n.

$$n! = 1 \cdot 2 \cdot 3 \cdots n$$

0! is defined to be 1, so that $n!$ is defined for all nonnegative integers n.

3. In an arithmetic sequence, the third term is 7 and the eighth term is 27. What is the tenth term in the sequence?

   (A) 35
   (B) 34
   (C) 33
   (D) 32
   (E) 31

*** Quick solution:** We can find the common difference of this arithmetic sequence with the computation

$$d = \frac{27-7}{8-3} = \frac{20}{5} = 4.$$

The ninth term is 27 + 4 = 31, the tenth term is 31 + 4 = 35, choice (A).

Remarks: (1) In an arithmetic sequence, you always add (or subtract) the same number to get from one term to the next. This can be done by moving forwards or backwards through the sequence.

(2) Questions about arithmetic sequences can easily be thought of as questions about lines and linear equations. We can identify terms of the sequence with points on a line where the x-coordinate is the term number and the y-coordinate is the term itself.

In the question above, since the third term of the sequence is 7, we can identify this term with the point (3,7). Since the eighth term of the sequence is 27, we can identify this with the point (8,27). Note that the common difference d is just the slope of the line that passes through these two points, i.e. $d = \frac{27-7}{8-3} = 4$.

Definition: An **arithmetic sequence** is a sequence of numbers such that the difference $\boldsymbol{d}$ between consecutive terms is constant. The number $\boldsymbol{d}$ is called the **common difference** of the arithmetic sequence.

Example of an arithmetic sequence: –1, 3, 7, 11, 15, 19, 23, 27, 31, 35,…

In this example the common difference is $d = 3 - (-1) = 4$.

Note that this is the same arithmetic sequence given in this question.

Arithmetic sequence formula: $\boldsymbol{a_n = a_1 + (n-1)d}$

In the above formula, a_n is the nth term of the sequence. For example, a_1 is the first term of the sequence.

Note: In the arithmetic sequence –1, 3, 7, 11, 15, 19, 23, 27, 31, 35,… we have that $a_1 = -1$ and $d = 4$. Therefore

$$a_n = -1 + (n-1)(4) = -1 + 4n - 4 = -5 + 4n.$$

It follows that $a_{10} = -5 + 4(10) = -5 + 40 = 35$, choice (A)**.**

Solution using the arithmetic sequence formula Substituting 3 in for n and 7 in for a_n into the arithmetic sequence formula gives us $7 = a_1 + 2d$.

Similarly, substituting 8 in for n and 27 in for a_n into the arithmetic sequence formula gives us $27 = a_1 + 7d$.

So we solve the following system of equations to find d.

$$\begin{array}{r} 27 = a_1 + 7d \\ \underline{\;\;7 = a_1 + 2d} \\ 20 = \qquad 5d \end{array}$$

The last equation comes from subtraction. We now divide each side of this last equation by 5 to get $d = 4$.

Finally, we add 4 to 27 twice to get $27 + 4(2) = 35$, choice (A).

Remarks: (1) We used the elimination method to find d here. This is usually the quickest way to solve a system of linear equations on this test.

(2) Once we have that $d = 4$, we can substitute this into either of the original equations to find a_1. For example, we have $7 = a_1 + 2(4)$, so that $a_1 = 7 - 8 = -1$.

4. A deposit of \$800 is made into an account that earns 2% interest compounded annually. If no additional deposits are made, how many years will it take until there is \$990 in the account?

(A) 9
(B) 10
(C) 11
(D) 12
(E) 13

* We use the formula $A = P(1+r)^t$ for interest compounded annually. We are given that P = 800, r = 0.02, A = 990, and we want to find t. So we have $990 = 800(1.02)^t$. We can now proceed in 2 ways.

Method 1 – Starting with choice (C): We start with choice (C) and substitute 11 in for t to get $800(1.02)^{11} \approx 994.7$. So the answer is (C).

Note that $800(1.02)^{10} \approx 975$, and this is too small.

Method 2 – Algebraic solution: We divide each side of the equation by 800 to get $1.2375 = (1.02)^t$. We then take the natural logarithm of each side to get $\ln 1.2375 = \ln(1.02)^t$. We can now use a basic law of logarithms to bring the t out in front of 1.02 (see the last row of the table below). We get $\ln 1.2375 = t \ln 1.02$.

Finally we use our calculator to divide $\ln 1.2375$ by $\ln 1.02$ to get $t \approx 10.76$. So it will take 11 years to get \$990 in the account, choice (C).

Laws of Logarithms: Here is a review of the basic laws of logarithms.

Law	Example
$\log_b 1 = 0$	$\log_2 1 = 0$
$\log_b b = 1$	$\log_6 6 = 1$
$\log_b x + \log_b y = \log_b(xy)$	$\log_5 7 + \log_5 2 = \log_5 14$
$\log_b x - \log_b y = \log_b(\frac{x}{y})$	$\log_3 21 - \log_3 7 = \log_3 3 = 1$
$\log_b x^n = n\log_b x$	$\log_8 3^5 = 5\log_8 3$

More general interest formula: For interest compounded n times a year we use the formula $A = P\left(1+\frac{r}{n}\right)^{nt}$ where n is the number of compoundings per year. For example if the interest is being compounded annually (once per year), then n = 1, and we get the formula used in the solution above. Other common examples are semiannually (n = 2), quarterly (n = 4), and monthly ($n = 12$).

LEVEL 1: ALGEBRA AND FUNCTIONS

5. $w(\frac{3}{5t} - \frac{1}{u}) =$

(A) $\frac{2}{5tw-uw}$

(B) $\frac{3uw-5tw}{5ut}$

(C) $\frac{2w}{5tu}$

(D) $\frac{2w}{5t-u}$

(E) $\frac{3w}{5tu}$

Solution by picking numbers: Let's let $w = 5$, $t = 2$ and $u = 10$. Then $w\left(\frac{3}{5t} - \frac{1}{u}\right) = 5\left(\frac{3}{10} - \frac{1}{10}\right) = 5\left(\frac{2}{10}\right) = \mathbf{1}$. Put a nice big, dark circle around **1** so you can find it easily later. We now substitute our values for w, t and u into each answer choice.

(A) $\frac{2}{50-50}$ = undefined

(B) $\frac{150-50}{100} = 1$

(C) $\frac{10}{100} = 0.1$

(D) $\frac{10}{10-10}$ = undefined

(E) $\frac{15}{100} = 0.15$

Since A, C, D and E are incorrect we can eliminate them. Therefore the answer is choice (B)**.**

*** Algebraic solution:** We have $w\left(\frac{3}{5t} - \frac{1}{u}\right) = w\left(\frac{3u}{5ut} - \frac{5t}{5ut}\right) = \frac{3uw-5tw}{5ut}$.

This is choice (B).

Notes: (1) To get from the first expression to the second we note that the the **least common denominator** is $5ut$. Since $\frac{3}{5t}$ already has $5t$ in the denominator we only need to multiply the denominator and numerator by u to get $\frac{3}{5t} \cdot \frac{u}{u} = \frac{3u}{5ut}$. Similarly, since $\frac{1}{u}$ already has u in the denominator we only need to multiply the denominator and numerator by $5t$ to get $\frac{1}{u} \cdot \frac{5t}{5t} = \frac{5t}{5ut}$.

(2) To get from the second expression to the third we first rewrite $\frac{3u}{5ut} - \frac{5t}{5ut}$ as $\frac{3u-5t}{5ut}$. We then distribute the w to get $\frac{3uw-5tw}{5ut}$.

6. If $2x + 3y = 5$, $2y + z = 3$, and $x + 5y + z = 3$, then $x =$

(A) 1
(B) 3
(C) 4
(D) 5
(E) 7

*** Solution by performing simple operations:** We add the first two equations and subtract the second equation to get $x = 5 + 3 - 3 = 5$, choice (D).

Computations in detail: We add the first two equations:

$$\begin{array}{r} 2x + 3y = 5 \\ \underline{ 2y + z = 3} \\ 2x + 5y + z = 8 \end{array}$$

We then subtract the third equation from this result:

$$\begin{array}{r} 2x + 5y + z = 8 \\ \underline{x + 5y + z = 3} \\ x = 5 \end{array}$$

Solution using Gauss-Jordan reduction: Push the MATRIX button, scroll over to EDIT and then select [A] (or press 1). We will be inputting a 3×4 matrix, so press 3 ENTER 4 ENTER. Then enter the numbers 2, 3, 0 and 5 for the first row, 0, 2, 1 and 3 for the second row, and 1, 5, 1 and 3 for the third row.

Now push the QUIT button (2ND MODE) to get a blank screen. Press MATRIX again. This time scroll over to MATH and select rref((or press B). Then press MATRIX again and select [A] (or press 1) and press ENTER.

The display will show the following.

$$\begin{matrix} [\,[1\ 0\ 0 & 5 &] \\ \ [0\ 1\ 0 & -1.67 &] \\ \ [0\ 0\ 1 & 6.33 &]\,] \end{matrix}$$

The first line is interpreted as $x = 5$, choice (D).

Notes: (1) In the first paragraph of this solution we created the **augmented matrix** for the system of equations. This is simply an array of numbers which contains the coefficients of the variables together with the right hand sides of the equations.

(2) In the second paragraph we put the matrix into **reduced row echelon form** (rref). In this form we can read off the solution to the original system of equations.

Warning: Be careful to use the rref(button (2 r's), and not the ref(button (which has only one r).

7. If $\sqrt{7b^3} = 3.27$, then b =

   (A) .43
   (B) 1.15
   (C) 1.53
   (D) 1.76
   (E) 12.66

*** Algebraic/calculator solution:** We square each side of the equation to get $7b^3 = 10.6929$. We then divide each side of this last equation by 7 to get $b^3 \approx 1.5276$. Finally, we take the cube root of each side of the last equation to get $b \approx 1.15$, choice (B).

Notes: (1) To take a cube root in your calculator you can either use the cube root function found in the MATH menu, or raise the number to the $\frac{1}{3}$ power. In this example you would type 1.5276 ^ (1 / 3) ENTER, or even better just use the calculator's previous answer and type ^ (1 / 3) ENTER.

(2) When possible try to get in the habit of using the calculator's previous answer instead of retyping decimal approximations. For example, this problem can be solved by pressing the following sequence of buttons:

3.27 ^ 2 / 7 ENTER ^ (1 / 3) ENTER

8. If $k(x) = \frac{x^2-1}{x+2}$ and $h(x) = \ln x^2$, then $k\big(h(e)\big)$ =

   (A) 0.12
   (B) 0.50
   (C) 0.51
   (D) 0.75
   (E) 1.25

*** Quick solution:** $h(e) = \ln e^2 = 2$. So $k(h(e)) = k(2) = \frac{2^2-1}{2+2} = 0.75$, choice (D).

Notes: (1) We first substituted e into the function h to get 2. We then substituted 2 into the function k to get 0.75.

(2) The word "logarithm" just means "exponent."

(3) The equation $y = \log_b x$ can be read as "y is the exponent when we rewrite x with a base of b." In other words we are raising b to the power y. So the equation can be written in exponential form as $x = b^y$.

(4) There are several ways to compute $\ln e^2$.

<u>Method 1</u>: Simply use your calculator.

<u>Method 2</u>: Recall that the functions e^x and $\ln x$ are inverses of each other. This means that $e^{\ln x} = x$ and $\ln e^x = x$. Substituting $x = 2$ into the second equation gives the desired result.

<u>Method 3</u>: Remember that $\ln x = \log_e x$. So we can rewrite the equation $y = \ln e^2$ in exponential form as $e^y = e^2$. So $y = 2$.

<u>Method 4</u>: Recall that $\ln e = 1$. We have $\ln e^2 = 2\ln e = 2(1) = 2$. Here we have used the last law in the table at the end of the solution to problem 4.

(5) The base b of a logarithm must satisfy $b > 0$ and $b \neq 1$.

9. If –7 and 5 are both zeros of the polynomial $q(x)$, then a factor of $q(x)$ is

   (A) $x^2 - 35$
   (B) $x^2 + 35$
   (C) $x^2 + 2x + 35$
   (D) $x^2 - 2x + 35$
   (E) $x^2 + 2x - 35$

*** Algebraic solution:** $(x + 7)$ and $(x - 5)$ are both factors of $q(x)$. Therefore so is $(x + 7)(x - 5) = x^2 + 2x - 35$, choice (E).

Note: There are several ways to multiply two binomials. One way familiar to many students is by FOILing. If you are comfortable with the method of FOILing you can use it here, but an even better way is to use the same algorithm that you already know for multiplication of whole numbers.

$$\begin{array}{r} x + 7 \\ \underline{x - 5} \\ -5x - 35 \\ \underline{x^2 + 7x + 0} \\ x^2 + 2x - 35 \end{array}$$

What we did here is mimic the procedure for ordinary multiplication. We begin by multiplying –5 by 7 to get –35. We then multiply –5 by x to get $-5x$. This is where the first row under the first line comes from.

Next we put 0 in as a placeholder on the next line. We then multiply x by 7 to get $7x$. And then we multiply x by x to get x^2. This is where the second row under the first line comes from.

Now we add the two rows to get $x^2 + 2x - 35$.

Solution by starting with choice (C): We are looking for the expression that gives 0 when we substitute in –7 and 5 for x.

Starting with choice (C) we have $5^2 + 2(5) + 35 = 70$. So we eliminate choice (C).

For choice (D) we have $5^2 - 2(5) + 35 = 50$. So we eliminate choice (D).

For choice (E) we have $5^2 + 2(5) - 35 = 0$ and $(-7)^2 + 2(-7) - 35 = 0$. So the answer is (E).

Notes: (1) c is a zero of a function $f(x)$ if $f(c) = 0$. For example, 5 is a zero of $x^2 + 2x - 35$ because $5^2 + 2(5) - 35 = 0$.

(2) A **polynomial** has the form $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ where a_0, $a_1, \ldots, a_n$ are real numbers. For example, $x^2 + 2x - 35$ is a polynomial.

(3) $p(c) = 0$ if and only if $x - c$ is a factor of the polynomial $p(x)$.

10. If $g(x) = x^2 - 3$ and $g\big(f(2)\big) = -2$, then $f(x)$ could be

(A) $x^3 - x^2 + x - 3$
(B) $x^3 - x^2 - 3$
(C) $x^3 + x - 3$
(D) $x^2 + x - 3$
(E) $x + 3$

*** Solution by starting with choice (C):** We start with choice (C) and guess that $f(x) = x^3 + x - 3$. We then have $f(2) = 2^3 + 2 - 3 = 7$ and $g(f(2)) = g(7) = 7^2 - 3 = 46$. This is incorrect so we can eliminate choice (C).

Let's try choice (B) next and guess that $f(x) = x^3 - x^2 - 3$. It follows that $f(2) = 2^3 - 2^2 - 3 = 1$ and so $g\big(f(2)\big) = g(1) = 1^2 - 3 = -2$. This is correct. So the answer is choice (B).

11. If $5x^2 - 2x + 3 = \frac{2}{7}(ax^2 + bx + c)$, then $a + b + c =$

    (A) 15
    (B) 17
    (C) 19
    (D) 21
    (E) 23

* Letting $x = 1$, the left hand side of the equation is $5(1)^2 - 2(1) + 3 = 6$ and the right hand side is $\frac{2}{7}(a(1)^2 + b(1) + c) = \frac{2}{7}(a + b + c)$. So we have that $\frac{2}{7}(a + b + c) = 6$, and so $a + b + c = 6(\frac{7}{2}) = 21$, choice (D).

12. If $y = \frac{2}{3}(x + 7)$ and $z = y + 212$, then which of the following expresses z in terms of x ?

    (A) $z = \frac{2}{3}(x + 219)$

    (B) $z = \frac{2}{3}(x + 205)$

    (C) $z = \frac{2}{3}(x - 7) + 212$

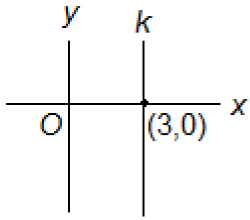
    (D) $z = \frac{2}{3}(x + 7) - 212$

    (E) $z = \frac{2}{3}(x + 7) + 212$

* Simply substitute $\frac{2}{3}(x + 7)$ in for y in the second equation to get $z = \frac{2}{3}(x + 7) + 212$, choice (E).

Remark: This problem can also be solved by picking a number for x. I leave it to the reader to solve the problem this way.

LEVEL 1: GEOMETRY



13. An equation of line k in the figure above is

(A) $x = 3$
(B) $y = 3$
(C) $x = 0$
(D) $y = x + 3$
(E) $x + y = 3$

* A vertical line has equation $x = c$, where c is the x-coordinate of ANY point on the line. So the answer is $x = 3$, choice (A).

Note: Any equation of the form $y = a$ for some real number a is a horizontal line. Any equation of the form $x = c$ for some real number c is a vertical line. Horizontal lines have a slope of 0 and vertical lines have no slope (or to be more precise, **undefined** slope or **infinite** slope).

14. What is the distance between the points $(-2,7)$ and $(3,-2)$

(A) 14
(B) $\sqrt{106}$
(C) $\sqrt{26}$
(D) $\frac{9}{5}$
(E) $\frac{5}{9}$

*** Solution using the distance formula:**

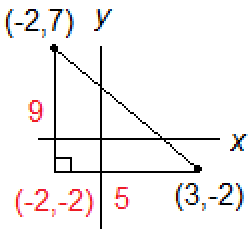
$$d = \sqrt{\left(3-(-2)\right)^2 + (-2-7)^2} = \sqrt{5^2 + (-9)^2} = \sqrt{25+81} = \sqrt{106}$$

This is choice (B).

Note: The distance between the points (s, t) and (u, v) is

$$d = \sqrt{(u-s)^2 + (v-t)^2}$$

Solution using the Pythagorean Theorem: We plot the two points and form a right triangle



The legs of the triangle have lengths 7 – (–2) = 9 and 3 – (–2) = 5. By the Pythagorean Theorem, the hypotenuse of the triangle has length

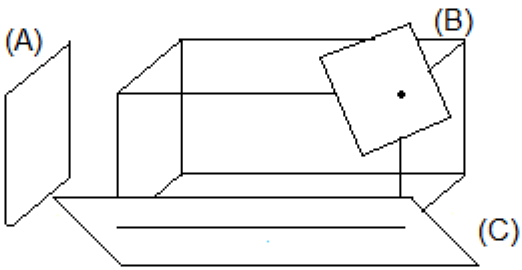
$\sqrt{9^2+5^2} = \sqrt{106}$, choice (B).

15. The intersection of a plane with a rectangular solid CANNOT be

(A) empty
(B) a point
(C) a line
(D) an ellipse
(E) a triangle

*** Solution by process of elimination:** If the plane is parallel to the rectangular solid the intersection can be empty. So we can eliminate choice (A). The plane can also touch a single vertex or a single edge of the rectangular solid, so we can eliminate choices (B) and (C). A diagonal slice through the solid can result in a triangle. So we can eliminate choice (E) and the answer is choice (D).

Visual explanation: The figure below shows a rectangular solid and three planes – one with empty intersection (A), one that intersects the solid in a point (B), and one that intersects the solid in a line (C). Can you draw a picture showing an intersection in a triangle?



16. What is the surface area of a cube with a volume of 125 in^3

(A) 5 in^2
(B) 25 in^2
(C) 75 in^2
(D) 120 in^2
(E) 150 in^2

* The length of an edge of the cube is $\sqrt[3]{125} = 5$ in. So the surface area of the cube is $6 \cdot 5^2 = 6 \cdot 25 = 150$ in^2, choice (E).

Some formulas: The **volume of a rectangular solid** is

$$\boldsymbol{V = lwh,}$$

where l, w and h are the length, width and height of the rectangular solid, respectively.

In particular, the **volume of a cube** is $\boldsymbol{V = s^3}$ where s is the length of a side of the cube.

The **surface area of a rectangular solid** is just the sum of the areas of all 6 faces. The formula is

$$\boldsymbol{A = 2lw + 2lh + 2wh}$$

where l, w and h are the length, width and height of the rectangular solid, respectively.

In particular, the **surface area of a cube** is

$$\boldsymbol{A = 6s^2}$$

where s is the length of a side of the cube.

17. In the rectangular coordinate system the point $P(a, b)$ is moved to the new point $Q(5a, 5b)$. If the distance between point Q and the origin is k, what is the distance between point P and the origin?

(A) $\frac{k}{5}$

(B) k

(C) $\frac{5}{k}$

(D) $5k$

(E) $25k$

Solution by picking numbers: Let's let $a = 1$ and $b = 2$. Then the points are $P(1,2)$ and $Q(5,10)$ and $k = \sqrt{5^2 + 10^2} = \sqrt{125} \approx 11.180$. The distance between P and the origin is $\sqrt{1^2 + 2^2} = \sqrt{5} \approx$ **2.236**. Put a nice big, dark circle around **2.236** so you can find it easily later. We now substitute our value for k into each answer choice.

(A) $\frac{11.180}{5} \approx 2.236$

(B) $k \approx 11.180$

(C) $\frac{5}{k} \approx .447$

(D) $5k \approx 55.9$

(E) $25k \approx 279.5$

Since B, C, D and E are incorrect we can eliminate them. Therefore the answer is choice (A)**.**

*** Direct solution:** The distance between Q and the origin is

$$k = \sqrt{(5a)^2 + (10b)^2} = \sqrt{25a^2 + 100b^2}. = \sqrt{25(a^2 + b^2)}$$
$$= \sqrt{25}\sqrt{a^2 + b^2} = 5\sqrt{a^2 + b^2}$$

The distance between P and the origin is $\sqrt{a^2 + b^2} = \frac{k}{5}$, choice (A).

Remark: These distances can also be computed by plotting points, drawing right triangles, and using the Pythagorean Theorem. See the second solution in problem 14 for details.

18. Which of the following is an equation of the line with an x-intercept of (4,0) and a y-intercept of (0,–3) ?

(A) $y = \frac{3}{4}x + 4$

(B) $y = \frac{3}{4}x - 3$

(C) $y = -\frac{3}{4}x + 4$

(D) $y = -\frac{3}{4}x - 3$

(E) $y = \frac{4}{3}x - 3$

*** Solution by plugging in points:** We plug in the given points to eliminate answer choices. Since the point (0,–3) is on the line, when we substitute a 0 in for x we should get –3 for y.

(A) 4
(B) –3
(C) 4
(D) –3
(E) –3

So we can eliminate choices A and C.

Since the point (4,0) is on the line, when we substitute a 4 for x we should get 0 for y.

(B) $\frac{3}{4}(4) - 3 = 0$
(D) $-\frac{3}{4}(4) - 3 = -6$
(E) $\frac{4}{3}(4) - 3 \approx 2.33$

So we can eliminate choices (D) and (E), and the answer is choice (B).

Algebraic solution: We write an equation of the line in slope-intercept form. The slope of the line is $\frac{-3-0}{0-4} = \frac{3}{4}$. Since (0, –3) is on the line, we have $b = -3$. So an equation of the line is $y = \frac{3}{4}x - 3$, choice (B).

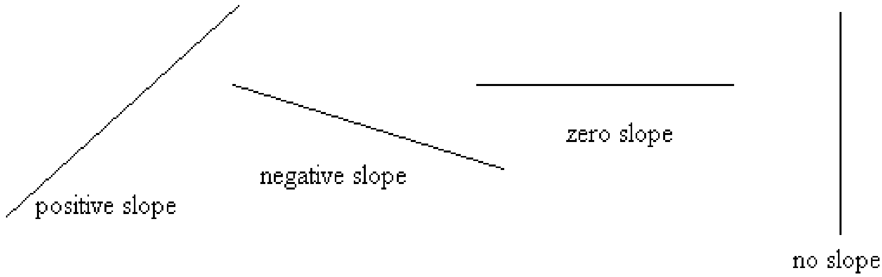
Remark: We could have also gotten the slope geometrically by plotting the two points, and noticing that to get from (0, –3) to (4,0) we need to travel up 3 units and right 4 units. So the slope is

$$m = \frac{rise}{run} = \frac{\mathbf{3}}{\mathbf{4}}.$$

Slope formula and linear equations:

$$\text{Slope} = m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

Note: Lines with positive slope have graphs that go upwards from left to right. Lines with negative slope have graphs that go downwards from left to right. If the slope of a line is zero, it is horizontal. Vertical lines have **no** slope, or **undefined** slope (this is different from zero slope).



The **slope-intercept form of an equation of a line** is $\boldsymbol{y = mx + b}$ where m is the slope of the line and b is the y-coordinate of the y-intercept, i.e. the point $(0, b)$ is on the line. Note that this point lies on the y-axis.

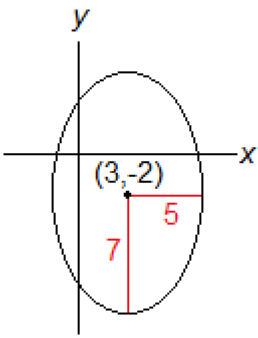
19. How long is the minor axis of the ellipse whose equation is $\frac{(x-3)^2}{25} + \frac{(y+2)^2}{49} = 1$?

    (A) 5
    (B) 7
    (C) 10
    (D) 14
    (E) 25

* The length of the minor axis is $2a = 2 \cdot 5 = 10$, choice (C).

Notes: (1) In the given equation $a^2 = 25$, so that $a = 5$, and the length of the minor axis is $2a = 2 \cdot 5 = 10$.

(2) Here is a picture of this ellipse. The line segment labelled with length 5 is half of the minor axis.



Ellipse facts: The standard form for an equation of an ellipse is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1.$$

The **center** of this ellipse is $(\boldsymbol{h}, \boldsymbol{k})$. $\boldsymbol{a}$ is the horizontal distance from the center of the ellipse to a vertex of the ellipse, and $\boldsymbol{b}$ is the vertical distance from the center of the ellipse to a vertex of the ellipse. The lengths of the two **axes** of the ellipse are $\boldsymbol{2a}$ and $\boldsymbol{2b}$. The larger of these two numbers is the **major axis** and the smaller of these two numbers is the **minor axis**.

20. Lines k and n are perpendicular and intersect at (0,0). If line n passes through the point (–3,1), then line k does NOT pass through which of the following points?

(A) (–2,–6)
(B) (–1, –2)
(C) (1,3)
(D) (3,9)
(E) (7,21)

* Line n has slope $\frac{1}{-3} = -\frac{1}{3}$ and therefore line k has slope 3. So an equation of line k is $y = 3x$. Since $-2 \neq 3(-1)$, the point (–1, –2) is NOT on line k. So the answer is choice (B).

Remarks: (1) Here we have used the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

(2) If the line j passes through the origin (the point (0, 0)) and the point (a, b) with $a \neq 0$, then the slope of line j is simply $\frac{b}{a}$.

(3) Perpendicular lines have slopes that are negative reciprocals of each other. The reciprocal of $-\frac{1}{3}$ is -3. The negative reciprocal of $-\frac{1}{3}$ is 3.

(4) Note that in answer choices A, C, D, and E, the y-coordinate of the point is 3 times the x-coordinate of the point.

LEVEL 1: PROBABILITY AND STATISTICS

21. The mean test grade of the 17 students in a geometry class was 66. When Johnny took a make-up test the next day, the mean test grade increased to 67. What grade did Johnny receive on the test?

(A) 83
(B) 84
(C) 85
(D) 86
(E) 87

Solution by changing averages to sums: We change the averages (or means) to sums using the formula

$$\text{Sum} = \text{Average} \cdot \text{Number}$$

We first average 17 numbers. Thus, the **Number** is 17. The **Average** is given to be 66. So the **Sum** of the 17 numbers is $66 \cdot 17 = 1122$.

When Johnny takes his make-up test, we average 18 numbers. Thus, the **Number** is 18. The new **Average** is given to be 67. So the **Sum** of the 18 numbers is $67 \cdot 18 = 1206$.

So Johnny received a grade of 1206 – 1122 = 84, choice (B).

22. In Bakerfield, 60% of the population own at least 1 cat. 20% of the cat owners in Bakerfield play the piano. If a resident of Bakerfield is selected at random, what is the probability that this person is a piano player that owns at least 1 cat?

    (A) 0.12
    (B) 0.15
    (C) 0.33
    (D) 0.37
    (E) 0.80

***** Let E be the event "owns at least 1 cat," and let F be the event "plays the piano." We are given $P(E) = .6$ and $P(F|E) = .2$. It follows that $P(E \cap F) = P(E) \cdot P(F|E) = (0.6)(0.2) = 0.12$, choice (A).

Notes: (1) To change a percent to a decimal, divide by 100, or equivalently move the decimal point two places to the left (adding zeros if necessary). Note that the number 60 has an "invisible" decimal point after the 0 (so that 60 = 60.). Moving the decimal to the left two places gives us .60 = .6.

(2) "60% of the population own at least 1 cat" is equivalent to "the probability that someone from the population owns a cat is .6." This was written above symbolically as $P(E) = .6$.

Similarly, "20% of the cat owners in Bakerfield play the piano" is equivalent to "the probability that someone from Bakerfield plays the piano **given** that this person owns a cat is .2." This was written above symbolically as $P(F|E) = .2$. Note that the symbol | is read "given," so that $P(F|E)$ is read "the probability of F given E.

(3) $E \cap F$ is read "the **intersection** of E and F. " It is the event consisting of the outcomes that are common to both E and F. In this problem a member of $E \cap F$ is a person from Bakerfield that owns at least 1 cat **and** plays the piano.

(4) $P(F|E)$ is called a **conditional probability**. The conditional probability formula is $\boldsymbol{P(E \cap F) = P(E) \cdot P(F|E)}$**.**

23. If $A = \{1,2,3,4,5,6,7,8,9,10\}$ and $B = \{3,6,9,12,15\}$, what is the mean of $A \cup B$?

    (A) 5.33
    (B) 5.82
    (C) 5.96
    (D) 6.24
    (E) 6.83

***** $A \cup B = \{1,2,3,4,5,6,7,8,9,10,12,15\}$. Therefore the mean of $A \cup B$ is $\frac{1+2+3+4+5+6+7+8+9+10+12+15}{12} = \frac{82}{12} \approx 6.83$, choice (E).

Notes: (1) $A \cup B$ is read "the **union** of A and B." It is the set consisting of the elements that are in A or B or both.

(2) The **average (arithmetic mean)** of a list of numbers is the sum of the numbers in the list divided by the quantity of the numbers in the list.

$$\textbf{Average} = \frac{\textbf{Sum}}{\textbf{Number}}$$