

**Advanced Course**

**28**

# ACT MATH LESSONS

to Improve Your  
Score in One Month

**By Dr. Steve Warner**

For Students Currently Scoring Above 25  
in ACT Math and Want to Score 36

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# **28 ACT Math**

## **Lessons to Improve Your Score in One Month**

**Advanced Course**

For Students Currently Scoring Above 25  
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Dr. Steve Warner



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# I N T R O D U C T I O N

## STUDYING FOR SUCCESS



his book was written specifically for the student currently scoring more than 25 in ACT math. Results will vary, but if you are such a student and you work through the lessons in this book, then you will see a substantial improvement in your score.

If your current ACT math score is below 25 or you discover that you have weaknesses in applying more basic techniques (such as the ones reviewed in the first lesson from this book), you may want to use *320 ACT Math Problems* until your score improves up to this level.

The book you are now reading is self-contained. Each lesson was carefully created to ensure that you are making the most effective use of your time while preparing for the ACT. It should be noted that a score of 30 can usually be attained without ever attempting a Level 5 problem. Readers currently scoring below a 30 on practice tests should not feel obligated to work on Level 5 problems the first time they go through this book.

The optional material in this book contains what I refer to as “Challenge” questions. Challenge questions may be more theoretical in nature and are much more difficult than anything that will ever appear on an ACT. These questions are for those students that really want an ACT math score of 36.

### **1. Using this book effectively**

- Begin studying at least three months before the ACT.
- Practice ACT math problems twenty minutes each day.
- Choose a consistent study time and location.

You will retain much more of what you study if you study in short bursts rather than if you try to tackle everything at once. So try to choose about a twenty-minute block of time that you will dedicate to ACT math each day. Make it a habit. The results are well worth this small time commitment. Some students will be able to complete each lesson within this twenty-minute block of time. If it takes you longer than twenty minutes to complete a lesson, you can stop when twenty minutes are up and then complete the lesson the following day. At the very least, take a nice long break, and then finish the lesson later that same day.

- Every time you get a question wrong, **mark it off, no matter what your mistake.**
- Begin each lesson by first redoing the problems from previous lessons on the same topic that you have marked off.
- If you get a problem wrong again, **keep it marked off.**

As an example, before you begin the third Number Theory lesson (Lesson 9), you should redo all the problems you have marked off from the first two Number Theory lessons (Lessons 1 and 5). Any question that you get right you can “unmark” while leaving questions that you get wrong marked off for the next time. If this takes you the full twenty minutes, that is okay. Just begin the new lesson the next day.

Note that this book often emphasizes solving each problem in more than one way. Please listen to this advice. The same question is never repeated on any ACT, and so the important thing is learning as many techniques as possible. Being able to solve any specific problem is of minimal importance. The more ways you have to solve a single problem, the more prepared you will be to tackle a problem you have never seen before, and the quicker you will be able to solve that problem. Also, if you have multiple methods for solving a single problem, then on the actual ACT when you “check over” your work you will be able to redo each problem in a different way. This will eliminate all “careless” errors on the actual exam. In this book the quickest solution to any problem will always be marked with an asterisk (\*).

## 2. Calculator use.

- Use a TI-84 or comparable calculator if possible when practicing and during the ACT.
- Make sure that your calculator has fresh batteries on test day.



- You may have to switch between DEGREE and RADIAN modes during the test. If you are using a TI-84 (or equivalent) calculator press the MODE button and scroll down to the third line when necessary to switch between modes.

Below are the most important things you should practice on your graphing calculator.

- Practice entering complicated computations in a single step.
- Know when to insert parentheses:
  - Around numerators of fractions
  - Around denominators of fractions
  - Around exponents
  - Whenever you actually see parentheses in the expression

### Examples:

We will substitute a 5 in for  $x$  in each of the following examples.

| <u>Expression</u>    | <u>Calculator computation</u> |
|----------------------|-------------------------------|
| $\frac{7x+3}{2x-11}$ | $(7*5 + 3)/(2*5 - 11)$        |
| $(3x-8)^{2x-9}$      | $(3*5 - 8)^(2*5 - 9)$         |

- Clear the screen before using it in a new problem. The big screen allows you to check over your computations easily.
- Press the **ANS** button (**2ND (-)**) to use your last answer in the next computation.
- Press **2ND ENTER** to bring up your last computation for editing. This is especially useful when you are plugging in answer choices, or guessing and checking.
- You can press **2ND ENTER** over and over again to cycle backwards through all the computations you have ever done.
- Know where the  $\sqrt{\quad}$ ,  $\pi$ , and  $\wedge$  buttons are so you can reach them quickly.
- Change a decimal to a fraction by pressing **MATH ENTER ENTER**.
- Press the **MATH** button - in the first menu that appears you can take cube roots and  $n$ th roots for any  $n$ . Scroll right to **NUM** and you have **lcm()** and **gcd()**. Scroll right to **PRB** and you have **nPr**, **nCr**, and **!** to compute permutations, combinations and factorials very quickly.

- Know how to use the **SIN**, **COS** and **TAN** buttons as well as **SIN<sup>-1</sup>**, **COS<sup>-1</sup>** and **TAN<sup>-1</sup>**.

You may find the following graphing tools useful.

- Press the **Y=** button to enter a function, and then hit **ZOOM 6** to graph it in a standard window.
- Practice using the **WINDOW** button to adjust the viewing window of your graph.
- Practice using the **TRACE** button to move along the graph and look at some of the points plotted.
- Pressing **2ND TRACE** (which is really **CALC**) will bring up a menu of useful items. For example, selecting **ZERO** will tell you where the graph hits the  $x$ -axis, or equivalently where the function is zero. Selecting **MINIMUM** or **MAXIMUM** can find the vertex of a parabola. Selecting **INTERSECT** will find the point of intersection of 2 graphs.

### 3. Tips for taking the ACT

Each of the following tips should be used whenever you take a practice ACT as well as on the actual exam.

**Check your answers properly:** When you go back to check your earlier answers for careless errors *do not* simply look over your work to try to catch a mistake. This is usually a waste of time.

- When “checking over” problems you have already done, **always redo the problem from the beginning** without looking at your earlier work.
- If possible, use a different method than you used the first time.

For example, if you solved the problem by picking numbers the first time, try to solve it algebraically the second time, or at the very least pick different numbers. If you do not know, or are not comfortable with a different method, then use the same method, but do the problem from the beginning and do not look at your original solution. If your two answers do not match up, then you know that this is a problem you need to spend a little more time on to figure out where your error is.

This may seem time consuming, but that is okay. It is better to spend more time checking over a few problems, than to rush through a lot of problems and repeat the same mistakes.

**Take a guess whenever you cannot solve a problem:** There is no guessing penalty on the ACT. Whenever you do not know how to solve a problem take a guess. Ideally you should eliminate as many answer choices as possible before taking your guess, but if you have no idea whatsoever do not waste time overthinking. Simply put down an answer and move on. You should certainly mark it off and come back to it later if you have time.

**Pace yourself:** After you have been working on a question for about 30 seconds you need to make a decision. If you understand the question and think that you can get the answer in another 30 seconds or so, continue to work on the problem. If you still do not know how to do the problem or you are using a technique that is going to take a long time, mark it off and come back to it later if you have time.

If you have eliminated at least one answer choice, or it is a grid-in, feel free to take a guess. But you still want to leave open the possibility of coming back to it later. Remember that every problem is worth the same amount. Do not sacrifice problems that you may be able to do by getting hung up on a problem that is too hard for you.

Now, after going through the test once, you can then go through each of the questions you have marked off and solve as many of them as you can. You should be able to spend 5 to 7 minutes on this, and still have 7 minutes left to check your answers. If there are one or two problems that you just cannot seem to get, let them go for a while. You can come back to them intermittently as you are checking over other answers.

# LESSON 1

## NUMBER THEORY

In this lesson we will be reviewing four very basic strategies that can be used to solve a wide range of ACT math problems in all topics and all difficulty levels. Throughout this book you should practice using these four strategies whenever it is possible to do so. You should also try to solve each problem in a more straightforward way.

### Start with the Middle Answer Choice

In many ACT math problems, you can get the answer simply by trying each of the answer choices until you find the one that works. Unless you have some intuition as to what the correct answer might be, then you should always start with the middle answer choice (C or H) as your first guess (an exception will be detailed in the next strategy below). The reason for this is simple. Answers are very often (but not always) given in increasing or decreasing order. So if the middle choice fails you can sometimes eliminate two of the other choices as well.

Try to answer the following question using this strategy. **Do not** check the solution until you have attempted this question yourself.

## LEVEL 2: NUMBER THEORY

1. Seven consecutive integers are listed in increasing order. If their sum is 350, what is the third integer in the list?
  - A. 45
  - B. 46
  - C. 47
  - D. 48
  - E. 49

### Solution

Begin by looking at choice C. If the third integer is 47, then the seven integers are 45, 46, 47, 48, 49, 50, and 51. Add these up in your calculator to get 336. This is too small. So we can eliminate choices A, B and C.

Since 336 is pretty far from 350 let's try choice E next. If the third integer is 49, then the seven integers are 47, 48, 49, 50, 51, 52, and 53. Add these up in your calculator to get 350. Therefore, the answer is choice E.

Before we go on, try to solve this problem in two other ways.

- (1) Algebraically (the way you would do it in school).
- (2) With a quick computation.

**Hint for (2):** In a set of consecutive integers, the average (arithmetic mean) and median are equal (see the optional material at the end of Lesson 20 for a proof of this).

### Solutions

**(1) Algebraic solution:** If we name the least integer  $x$ , then the seven integers are  $x, x + 1, x + 2, x + 3, x + 4, x + 5,$  and  $x + 6$ . So we have

$$\begin{aligned}x + (x + 1) + (x + 2) + (x + 3) + (x + 4) + (x + 5) + (x + 6) &= 350 \\7x + 21 &= 350 \\7x &= 329 \\x &= 47\end{aligned}$$

The third integer is  $x + 2 = 49$ , choice E.

**Important:** Always remember to check what the question is asking for before choosing your answer. Many students would accidentally choose choice C here as soon as they discovered that  $x = 47$ .

It is not a bad idea to underline the word "third" as you read the question. This may help you to avoid this kind of error.

\* **(2) Quick solution:** Divide 350 by 7 to get that the fourth integer is 50. Thus, the third integer is 49, choice E.

**Justification for the last solution:** Recall from the algebraic solution above that  $7x + 21 = 350$ . Thus,  $7(x + 3) = 350$ , and therefore  $x + 3 = \frac{350}{7} = 50$ . Finally, we have  $x + 2 = 50 - 1 = 49$ .

Note that  $x + 3$  is the median of the seven integers (the fourth integer in the list),  $\frac{350}{7}$  is the average of the seven integers, and these two quantities are equal. See Lesson 20 for more details.

### When NOT to Start with the Middle Answer Choice

If the word **least** appears in the problem, then start with the smallest number as your first guess. Similarly, if the word **greatest** appears in the problem, then start with the largest number as your first guess.

Try to answer the following question using this strategy. **Do not** check the solution until you have attempted this question yourself.

## LEVEL 3: NUMBER THEORY

2. What is the largest positive integer value of  $k$  for which  $7^k$  divides  $147^{15}$  ?
- F. 3
  - G. 7
  - H. 15
  - J. 28
  - K. 30

### Solution

\*Pull out your calculator. Since the question has the word “**largest**” in it, we will start with the largest answer choice which is choice E, and we divide  $147^{15}$  by  $7^{30}$ . We type  $147^{15} / 7^{30}$  into our calculator and the output is 14,348,907. Since this is an integer, the answer is choice K.

Note that all five answer choices give an integer, but 30 is the largest positive integer that works.

Before we go on, try to solve this problem directly (without using the answer choices).

### Solution

The prime factorization of 147 is  $147 = 3 \cdot 7^2$ . Therefore,

$$147^{15} = (3 \cdot 7^2)^{15} = 3^{15}(7^2)^{15} = 3^{15} 7^{30}.$$

So  $7^{30}$  divides  $147^{15}$ , but  $7^{31}$  does not. Thus, the answer is choice **E**.

**Note:** Prime factorizations will be reviewed in Lesson 5.

### Take a Guess

Sometimes the answer choices themselves cannot be substituted in for the unknown or unknowns in the problem. But that does not mean that you cannot guess your own numbers. Try to make as reasonable a guess as possible, but do not over think it. Keep trying until you zero in on the correct value.

Try to answer the following question using this strategy. **Do not** check the solution until you have attempted this question yourself.

## LEVEL 3: NUMBER THEORY

3. Dana has pennies, nickels and dimes in her pocket. The number of dimes she has is three times the number of nickels, and the number of nickels she has is 2 more than the number of pennies. Which of the following could be the total number of coins in Dana's pocket?
- A. 14
  - B. 15
  - C. 16
  - D. 17
  - E. 18

### Solution

\* Let's take a guess and say that Dana has 3 pennies. It follows that she has  $3 + 2 = 5$  nickels, and  $(3)(5) = 15$  dimes. So the total number of coins is  $3 + 5 + 15 = 23$ . This is too many. So let's guess that Dana has 2 pennies. Then she has  $2 + 2 = 4$  nickels, and  $(3)(4) = 12$  dimes for a total of  $2 + 4 + 12 = 18$  coins. Thus, the answer is choice **E**.

Before we go on, try to solve this problem the way you might do it in school.

### Solution

If we let  $x$  represent the number of pennies, then the number of nickels is  $x + 2$ , and the number of dimes is  $3(x + 2)$ . Thus, the total number of coins is

$$x + (x + 2) + 3(x + 2) = x + x + 2 + 3x + 6 = 5x + 8.$$

So some possible totals are 13, 18, 23,... which we get by substituting 1, 2, 3,... for  $x$ . Substituting 2 in for  $x$  gives 18 which is answer choice E.

**Warning:** Many students incorrectly interpret “three times the number of nickels” as  $3x + 2$ . This is not right. The number of nickels is  $x + 2$ , and so “three times the number of nickels” is  $3(x + 2) = 3x + 6$ .

### Pick a Number

A problem may become much easier to understand and to solve by substituting a specific number in for a variable. Just make sure that you choose a number that satisfies the given conditions.

Here are some guidelines when picking numbers.

- (1) Pick a number that is simple but not too simple. In general, you might want to avoid picking 0 or 1 (but 2 is usually a good choice).
- (2) Try to avoid picking numbers that appear in the problem.
- (3) When picking two or more numbers try to make them all different.
- (4) Most of the time picking numbers only allows you to eliminate answer choices. So do not just choose the first answer choice that comes out to the correct answer. If multiple answers come out correct you need to pick a new number and start again. But you only have to check the answer choices that have not yet been eliminated.
- (5) If there are fractions in the question a good choice might be the least common denominator (lcd) or a multiple of the lcd.
- (6) In percent problems choose the number 100.
- (7) If your first attempt does not eliminate 4 of the 5 choices, try to choose a number that's of a different “type.” Here are some examples of types:



- (a) A positive integer greater than 1.
  - (b) A positive fraction (or decimal) between 0 and 1.
  - (c) A negative integer less than  $-1$ .
  - (d) A negative fraction (or decimal) between  $-1$  and 0.
- (8) If you are picking pairs of numbers, try different combinations from (7). For example, you can try two positive integers greater than 1, two negative integers less than  $-1$ , or one positive and one negative integer, etc.

Remember that these are just guidelines and there may be rare occasions where you might break these rules. For example, sometimes it is so quick and easy to plug in 0 and/or 1 that you might do this even though only some of the answer choices get eliminated.

Try to answer the following question using this strategy. **Do not** check the solution until you have attempted this question yourself.

## LEVEL 4: NUMBER THEORY

4.  $n$  is a two-digit number whose units digit is 3 times its tens digit, which of the following statements must be true?
- F.  $n$  is less than 15
  - G.  $n$  is greater than 30
  - H.  $n$  is a multiple of 3
  - J.  $n$  is a multiple of 10
  - K.  $n$  is a multiple of 13

### Solution

Let's choose a value for  $n$ , say  $n = 13$ . Notice that we chose a number whose units digit is 3 times its tens digit.

Now let's check if each answer choice is true or false.

- F. True
- G. False
- H. False
- J. False
- K. True

Since G, H, and J are each false we can eliminate them. Let's choose a new value for  $n$ , say  $n = 26$ . Let's check if each of choices F and K is true or false with this new value for  $n$ .

**F.** False

**K.** True

Choice F does not give the correct answer this time so we can eliminate it. Thus, the answer is choice **K**.

**Notes:** (1) When we chose our first number we needed to check **every** answer choice. A common mistake would be to choose answer choice F because it was the first one to come out true. When we choose our second number we only have to check the answer choices that haven't yet been eliminated.

(2) There are only 3 possibilities for  $n$ : 13, 26, and 39. Note that each of these 3 numbers is a multiple of 13.

You're doing great! Let's just practice a bit more. Try to solve each of the following problems by using one of the four strategies you just learned. Then, if possible, solve each problem another way. The answers to these problems, followed by full solutions are at the end of this lesson. **Do not** look at the answers until you have attempted these problems yourself. Please remember to mark off any problems you get wrong.

## LEVEL 1: NUMBER THEORY

5. Which of the following numbers has the greatest value?
- A.  $0.\overline{7}$
  - B. 0.7
  - C. 0.77
  - D. 0.777
  - E. 0.7777
6. The square root of a specific number is approximately 7.6315. The specific number is between what 2 integers?
- F. 2 and 3
  - G. 3 and 5
  - H. 7 and 15
  - J. 14 and 29
  - K. 49 and 63

**LEVEL 2: NUMBER THEORY**

7. What is the least common denominator of the fractions  $\frac{3}{10}$ ,  $\frac{2}{45}$ , and  $\frac{5}{27}$ ?
- A. 54
  - B. 270
  - C. 450
  - D. 2430
  - E. 12,150

**LEVEL 3: NUMBER THEORY**

8. What positive number when divided by its reciprocal has a result of  $\frac{9}{16}$ ?
- F.  $\frac{8}{3}$
  - G.  $\frac{3}{8}$
  - H.  $\frac{4}{3}$
  - J.  $\frac{3}{16}$
  - K.  $\frac{3}{4}$
9. A group of friends will rent out a hotel for \$1800 for a party. The cost of the hotel will be equally distributed among the friends who plan to attend the party. The current cost per person will increase by \$15 if 6 of the friends decide not to attend the party. How many friends are currently planning to attend the party?
- A. 10
  - B. 20
  - C. 24
  - D. 30
  - E. 42

## LEVEL 4: NUMBER THEORY

10. If  $c$  is a positive odd integer and  $d$  is a positive even integer, then  $[(+7)(-7)]^{cd}$
- F. negative and even
  - G. negative and odd
  - H. positive and even
  - J. positive and odd
  - K. zero
11. For every negative real value of  $a$ , all of the following are true EXCEPT:
- A.  $|3a| > 0$
  - B.  $a^7 < 0$
  - C.  $5a < 0$
  - D.  $a - |a| = 0$
  - E.  $4a - 2a^2 < 0$
12. A ball is dropped from 567 centimeters above the ground and after the fourth bounce it rises to a height of 7 centimeters. If the height to which the ball rises after each bounce is always the same fraction of the height reached on its previous bounce, what is this fraction?
- F.  $\frac{1}{81}$
  - G.  $\frac{1}{27}$
  - H.  $\frac{1}{9}$
  - J.  $\frac{1}{3}$
  - K.  $\frac{1}{2}$

### Answers

- |      |      |       |
|------|------|-------|
| 1. E | 5. A | 9. D  |
| 2. K | 6. K | 10. J |
| 3. E | 7. B | 11. D |
| 4. K | 8. K | 12. J |

Full Solutions

8.

**Solution by starting with choice H:** Let's start with  $\frac{4}{3}$  as our first guess. The reciprocal of  $\frac{4}{3}$  is  $\frac{3}{4}$ , and when we divide  $\frac{4}{3}$  by  $\frac{3}{4}$ , we get  $\frac{16}{9}$ . This is not correct, but it is the reciprocal of what we are trying to get. So the answer is the reciprocal of  $\frac{4}{3}$ , which is  $\frac{3}{4}$ , choice **K**.

**Notes:** (1) The **reciprocal** of the fraction  $\frac{a}{b}$  is the fraction  $\frac{b}{a}$ . In other words, we get the reciprocal of the fraction by interchanging the number on top (the **numerator**) with the number on bottom (the **denominator**).

(2) We can divide  $\frac{4}{3}$  by  $\frac{3}{4}$  right in our TI-84 calculator by typing

$(4 / 3) / (3 / 4)$  ENTER MATH ENTER ENTER

The output will be  $\frac{16}{9}$ .

Pressing MATH ENTER ENTER at the end changes the decimal to a fraction.

(3) We can also do the computation by hand as follows:

$$\frac{4}{3} \div \frac{3}{4} = \frac{4}{3} \cdot \frac{4}{3} = \frac{4 \cdot 4}{3 \cdot 3} = \frac{16}{9}$$

(4) Let's also just confirm that choice K is the answer. I'll use the hand method, but you can also feel free to use your calculator.

$$\frac{3}{4} \div \frac{4}{3} = \frac{3}{4} \cdot \frac{3}{4} = \frac{3 \cdot 3}{4 \cdot 4} = \frac{9}{16}$$

\* **Algebraic solution:** Let  $x$  be the positive number. We are given that

$$x \div \frac{1}{x} = \frac{9}{16}$$

$$x \cdot \frac{x}{1} = \frac{9}{16}$$

$$x \cdot x = \frac{9}{16}$$

$$x^2 = \frac{9}{16}$$

$$x = \pm \sqrt{\frac{9}{16}} = \pm \frac{\sqrt{9}}{\sqrt{16}} = \pm \frac{3}{4}$$

Since we are given that  $x$  is a positive number,  $x = \frac{3}{4}$ , choice **K**.

9.

**\* Solution by starting with choice C:** Let's start with choice C and guess that 24 friends are planning to attend the party. Then they are each paying  $\frac{1800}{24} = 75$  dollars. If 6 friends decide not to go, then there will be  $24 - 6 = 18$  friends attending the party, and they would each have to pay  $\frac{1800}{18} = 100$  dollars. The increase is  $100 - 75 = 25$  dollars, too big.

Let's try choice D next and guess that 30 friends are planning to attend the party. Then they are each paying  $\frac{1800}{30} = 60$  dollars. If 6 friends decide not to go, then there will be  $30 - 6 = 24$  friends attending the party, and they would each have to pay  $\frac{1800}{24} = 75$  dollars. The increase is  $75 - 60 = 15$  dollars. This is correct, and so the answer is choice D.

**Algebraic solution:** Let  $n$  be the number of friends planning to attend the party. Then each friend will be paying  $\frac{1800}{n}$  dollars. If 6 friends were to decide not to attend the party, then the number of friends remaining would be  $n - 6$ , and each remaining friend would pay  $\frac{1800}{n-6}$  dollars. We are also given that the current cost would increase by \$15 in this case, and so we have

$$\frac{1800}{n-6} = \frac{1800}{n} + 15$$

If we multiply each side of the equation by  $n(n-6)$ , we get

$$\begin{aligned} 1800n &= 1800(n-6) + 15n(n-6) \\ 1800n &= 1800n - 10,800 + 15n^2 - 60n \\ 15n^2 - 60n - 10,800 &= 0 \\ n^2 - 4n - 720 &= 0 \\ (n-30)(n+24) &= 0 \\ n-30 = 0 \text{ or } n+24 = 0 \\ n = 30 \text{ or } n = -24 \end{aligned}$$

So  $n = 30$ , choice **D**.

**Note:** Do not worry too much if you have trouble understanding this algebraic solution. We will be reviewing all of this algebra later in this book.

10.

\* **Solution by picking numbers:** Let's let  $c = 1$  and  $d = 2$ . It follows that

$$[(+7)(-7)]^{cd} = [(+7)(-7)]^2 = 2,401$$

We now check if each answer choice is true or false:

|           |                   |       |
|-----------|-------------------|-------|
| <b>F.</b> | negative and even | False |
| <b>G.</b> | negative and odd  | False |
| <b>H.</b> | positive and even | False |
| <b>J.</b> | positive and odd  | True  |
| <b>K.</b> | zero              | False |

Since only choice J is true, the answer is **J**.

**Notes:** (1) The following describes what happens when you add and multiply various combinations of even and odd integers.

$$\begin{array}{ll} e + e = e & ee = e \\ e + o = o & eo = e \\ o + e = o & oe = e \\ o + o = e & oo = o \end{array}$$

For example, the product of two odd integers is odd ( $oo = o$ ).

(2) Observe that the behavior described in note (1) is independent of the choices of the integers themselves. All that matters is whether the integers are even or odd. It follows that we can choose *any* positive odd integer for  $c$ , and *any* positive even integer for  $d$ .

**Direct solution:** Since  $c$  is positive and odd, and  $d$  is positive and even, it follows that  $cd$  is positive and even.

So we are raising  $-49$  to a positive even power.

When we raise an integer to a positive even power, the result is always positive. So this narrows down the answer to choice H or J.

When we multiply two odd numbers together, the result is always odd. In particular, multiplying an odd number by itself over and over again will always give an odd result. So the answer is choice J.

11.

**Solution by picking numbers and starting with choice C:** We need to find a negative real value of  $a$  that makes one of the statements false.

Let's try  $a = -2$  and start with choice C. We have  $5(-2) = -10 < 0$ . So for this value of  $a$ , C is true.

Let's try D next. We have  $-2 - |-2| = -2 - 2 = -4 \neq 0$ . So D is false for this choice of  $a$ , and therefore the answer is **D**.

**Note:** (1) Let's plug  $a = -2$  into each answer choice just to see that the remaining answer choices are true.

- |    |  |       |
|----|--|-------|
| A. | $ 3(-2)  =  -6  = 6 > 0$               | True  |
| B. | $(-2)^7 = -128 < 0$                    | True  |
| C. | $5(-2) = -10 < 0$                      | True  |
| D. | $-2 -  -2  = -2 - 2 = -4 \neq 0$       | False |
| E. | $4(-2) - 2(-2)^2 = -8 - 2 \cdot 4 < 0$ | True  |

(2) Normally picking numbers can be used only for eliminating answer choices. In this case however the wording of the question allows us to choose the first choice that fails for a specific value of  $a$ .

**Solution by process of elimination:** The absolute value of any nonzero number is always positive. Since  $a$  is nonzero (it's negative), it follows that  $3a$  is nonzero, and so  $|3a| > 0$ . This eliminates A.

A negative number raised to an odd power is negative. This eliminates B.

The product of a positive number and a negative number is negative. Since 5 is positive and  $a$  is negative,  $5a < 0$ . This eliminates C.

$4a - 2a^2 = 2a(2 - a)$ . Since  $a$  is negative,  $2a$  is negative (by reasoning in last paragraph) and  $2 - a$  is positive (pos - neg = pos + pos = pos), and so the product satisfies  $2a(2 - a) < 0$  (again by the reasoning in the last paragraph). This eliminates E.

So the answer is **D**.

\* **Direct solution:** Let  $a$  be negative. Then, since  $a \neq 0$ ,  $|a|$  is positive. So  $a - |a|$  is negative (a negative number minus a positive number is the same as adding two negative numbers, and therefore the result is negative). In particular,  $a - |a| \neq 0$ . So the answer is **D**.

**Note:** We needed to find a single negative number  $a$  that makes choice D false. We actually just showed something much stronger than this. We showed that D is false for *all* negative values of  $a$ .



12.

\* **Solution by starting with choice H:** Let's begin with choice H. We divide 567 by 9 four times and get 0.0864197531 which is much too small. So we can eliminate choices F, G, and H. We next try choice J. If we divide 567 by 3 four times we get 7 so that the correct answer is J.

**Note:** We could have also multiplied 7 by 3 four times to get 567.

**An algebraic solution:** We want to solve the following equation.

$$567x^4 = 7$$

$$x^4 = \frac{1}{81}$$

$$x = \frac{1}{3}$$

Thus, the answer is choice J.

## About the Author

Dr. Steve Warner, a New York native, earned his Ph.D. at Rutgers University in Pure Mathematics in May, 2001. While a graduate student, Dr. Warner won the TA Teaching Excellence Award.



After Rutgers, Dr. Warner joined the Penn State Mathematics Department as an Assistant Professor. In September, 2002, Dr. Warner returned to New York to accept an Assistant Professor position at Hofstra University. By September 2007, Dr. Warner had received tenure and was promoted to Associate Professor. He has taught undergraduate and graduate courses in Precalculus, Calculus, Linear Algebra, Differential Equations, Mathematical Logic,

Set Theory and Abstract Algebra.

Over that time, Dr. Warner participated in a five year NSF grant, “The MSTP Project,” to study and improve mathematics and science curriculum in poorly performing junior high schools. He also published several articles in scholarly journals, specifically on Mathematical Logic.

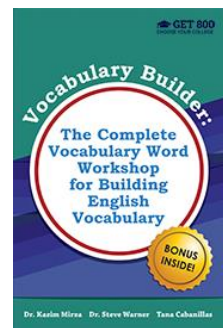
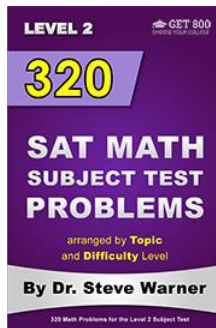
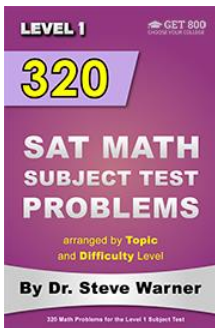
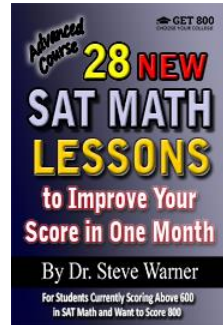
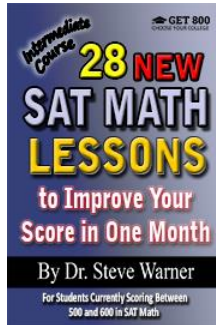
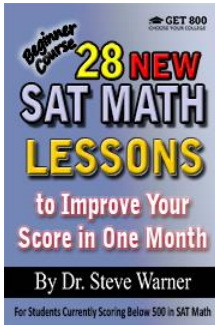
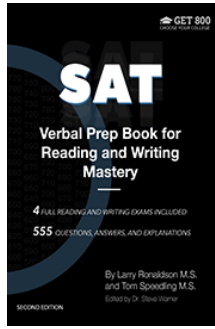
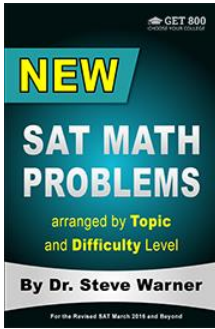
Dr. Warner has more than 15 years of experience in general math tutoring and tutoring for standardized tests such as the SAT, ACT and AP Calculus exams. He has tutored students both individually and in group settings.

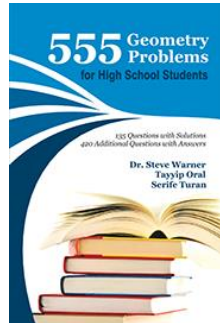
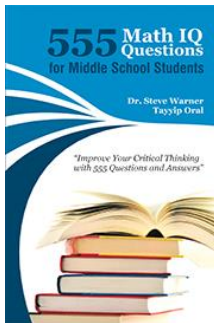
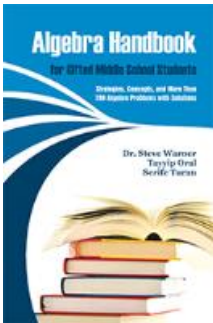
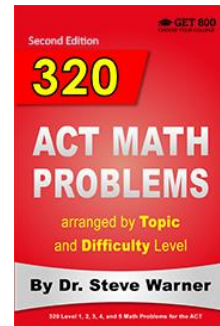
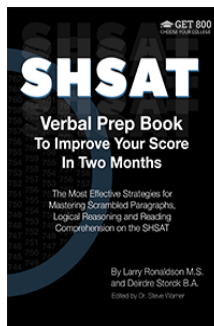
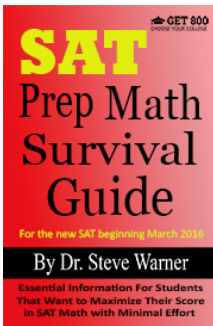
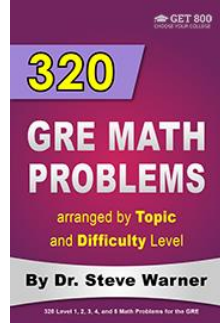
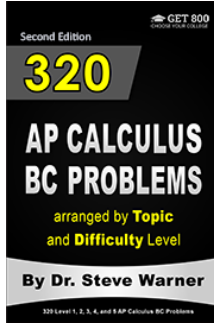
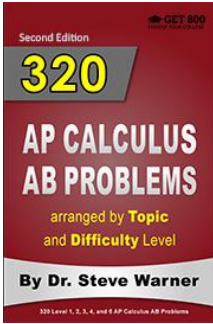
In February, 2010 Dr. Warner released his first SAT prep book “The 32 Most Effective SAT Math Strategies,” and in 2012 founded Get 800 Test Prep. Since then Dr. Warner has written books for the SAT, ACT, SAT Math Subject Tests and AP Calculus exams.

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