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Advanced Course

28 SAT MATH LESSONS

2017 Edition

to Improve Your Score in One Month

By Dr. Steve Warner

For Students Currently Scoring Above 600 in SAT Math and Want to Score 800

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Third Edition

BOOKS FROM THE GET 800 COLLECTION FOR COLLEGE BOUND STUDENTS

28 SAT Math Lessons to Improve Your Score in One Month
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INTRODUCTION

STUDYING FOR SUCCESS

This book was written specifically for the student currently scoring more than 600 in SAT math. Results will vary, but if you are such a student and you work through the lessons in this book, then you will see a substantial improvement in your score.

If your current SAT math score is below 600 or you discover that you have weaknesses in applying more basic techniques (such as the ones reviewed in the first lesson from this book), you may want to go through the intermediate course before completing this one.

The book you are now reading is self-contained. Each lesson was carefully created to ensure that you are making the most effective use of your time while preparing for the SAT. It should be noted that a score of 700 can usually be attained without ever attempting a Level 5 problem. Readers currently scoring below a 700 on practice tests should not feel obligated to work on Level 5 problems the first time they go through this book.

The optional material in this book contains what I refer to as "Level 6" questions and "Challenge" questions. Level 6 questions are slightly more difficult than anything that is likely to appear on an actual SAT, but they are just like SAT problems in every other way. Challenge questions are theoretical in nature and are much more difficult than anything that will ever appear on an SAT. These two types of questions are for those students that really want an SAT math score of 800.

There are two math sections on the SAT: one where a calculator is allowed and one where it is not. I therefore recommend trying to solve as many problems as possible both with and without a calculator. If a calculator is required for a specific problem it will be marked with an asterisk (*).

1. Using this book effectively

- Begin studying at least three months before the SAT
- Practice SAT math problems twenty minutes each day
- Choose a consistent study time and location

You will retain much more of what you study if you study in short bursts rather than if you try to tackle everything at once. So try to choose about a twenty minute block of time that you will dedicate to SAT math each day. Make it a habit. The results are well worth this small time commitment. Some students will be able to complete each lesson within this twenty minute block of time. If it takes you longer than twenty minutes to complete a lesson, you can stop when twenty minutes are up and then complete the lesson the following day. At the very least, take a nice long break, and then finish the lesson later that same day.

- Every time you get a question wrong, **mark it off, no matter what your mistake**.
- Begin each lesson by first redoing the problems from previous lessons on the same topic that you have marked off.
- If you get a problem wrong again, **keep it marked off**.

As an example, before you begin the third “Heart of Algebra” lesson (Lesson 9), you should redo all the problems you have marked off from the first two “Heart of Algebra” lessons (Lessons 1 and 5). Any question that you get right you can “unmark” while leaving questions that you get wrong marked off for the next time. If this takes you the full twenty minutes, that is okay. Just begin the new lesson the next day.

Note that this book often emphasizes solving each problem in more than one way. Please listen to this advice. The same question is never repeated on any SAT (with the exception of questions from the experimental sections) so the important thing is learning as many techniques as possible. Being able to solve any specific problem is of minimal importance. The more ways you have to solve a single problem the more prepared you will be to tackle a problem you have never seen before, and the quicker you will be able to solve that problem. Also, if you have multiple methods for solving a single problem, then on the actual SAT when you “check over” your work you will be able to redo each problem in a different way. This will eliminate all “careless” errors on the actual exam. In this book the quickest solution to any problem will always be marked with an asterisk (*).

2. Calculator use.

- Use a TI-84 or comparable calculator if possible when practicing and during the SAT.
- Make sure that your calculator has fresh batteries on test day.
- You may have to switch between DEGREE and RADIAN modes during the test. If you are using a TI-84 (or equivalent) calculator press the MODE button and scroll down to the third line when necessary to switch between modes.

Below are the most important things you should practice on your graphing calculator.

- Practice entering complicated computations in a single step.
- Know when to insert parentheses:
  - Around numerators of fractions
  - Around denominators of fractions
  - Around exponents
  - Whenever you actually see parentheses in the expression

Examples:

We will substitute a 5 in for x in each of the following examples.

Expression	Calculator computation
$\frac{7x+3}{2x-11}$	(7*5 + 3)/(2*5 – 11)
$(3x-8)^{2x-9}$	(3*5 – 8)^(2*5 – 9)

- Clear the screen before using it in a new problem. The big screen allows you to check over your computations easily.
- Press the **ANS** button (**2ND (-)**) to use your last answer in the next computation.
- Press **2ND ENTER** to bring up your last computation for editing. This is especially useful when you are plugging in answer choices, or guessing and checking.
- You can press **2ND ENTER** over and over again to cycle backwards through all the computations you have ever done.
- Know where the $\sqrt{\ }$, π, and **^** buttons are so you can reach them quickly.
- Change a decimal to a fraction by pressing **MATH ENTER ENTER**.

- Press the **MATH** button - in the first menu that appears you can take cube roots and nth roots for any n. Scroll right to **NUM** and you have **lcm(** and **gcd(**.
- Know how to use the **SIN**, **COS** and **TAN** buttons as well as **SIN^{-1}**, **COS^{-1}** and **TAN^{-1}.**

You may find the following graphing tools useful.

- Press the **Y=** button to enter a function, and then hit **ZOOM 6** to graph it in a standard window.
- Practice using the **WINDOW** button to adjust the viewing window of your graph.
- Practice using the **TRACE** button to move along the graph and look at some of the points plotted.
- Pressing **2ND TRACE** (which is really **CALC**) will bring up a menu of useful items. For example selecting **ZERO** will tell you where the graph hits the x-axis, or equivalently where the function is zero. Selecting **MINIMUM** or **MAXIMUM** can find the vertex of a parabola. Selecting **INTERSECT** will find the point of intersection of 2 graphs.

3. Tips for taking the SAT

Each of the following tips should be used whenever you take a practice SAT as well as on the actual exam.

Check your answers properly: When you go back to check your earlier answers for careless errors *do not* simply look over your work to try to catch a mistake. This is usually a waste of time.

- When "checking over" problems you have already done, **always redo the problem from the beginning** without looking at your earlier work.
- If possible use a different method than you used the first time.

For example, if you solved the problem by picking numbers the first time, try to solve it algebraically the second time, or at the very least pick different numbers. If you do not know, or are not comfortable with a different method, then use the same method, but do the problem from the beginning and do not look at your original solution. If your two answers do not match up, then you know that this is a problem you need to spend a little more time on to figure out where your error is.

This may seem time consuming, but that is okay. It is better to spend more time checking over a few problems, than to rush through a lot of problems and repeat the same mistakes.

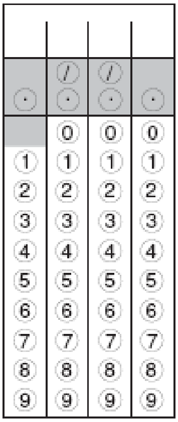
Take a guess whenever you cannot solve a problem: There is no guessing penalty on the SAT. Whenever you do not know how to solve a problem take a guess. Ideally you should eliminate as many answer choices as possible before taking your guess, but if you have no idea whatsoever do not waste time overthinking. Simply put down an answer and move on. You should certainly mark it off and come back to it later if you have time.

Pace yourself: After you have been working on a question for about 30 seconds you need to make a decision. If you understand the question and think that you can get the answer in another 30 seconds or so, continue to work on the problem. If you still do not know how to do the problem or you are using a technique that is going to take a long time, mark it off and come back to it later if you have time.

Feel free to take a guess. But you still want to leave open the possibility of coming back to it later. Remember that every problem is worth the same amount. Do not sacrifice problems that you may be able to do by getting hung up on a problem that is too hard for you.

Now, after going through the test once, you can then go through each of the questions you have marked off and solve as many of them as you can. You should be able to spend 5 to 7 minutes on this, and still have 7 minutes left to check your answers. If there are one or two problems that you just cannot seem to get, let them go for a while. You can come back to them intermittently as you are checking over other answers.

Grid your answers correctly: The computer only grades what you have marked in the bubbles. The space above the bubbles is just for your convenience, and to help you do your bubbling correctly.



Never mark more than one circle in a column or the problem will automatically be marked wrong. You do not need to use all four columns. If you do not use a column just leave it blank.

The symbols that you can grid in are the digits 0 through 9, a decimal point, and a division symbol for fractions. Note that there is no negative symbol. So answers to grid-ins *cannot* be negative. Also, there are only four slots, so you cannot get an answer such as 52,326.

Sometimes there is more than one correct answer to a grid-in question. Simply choose one of them to grid-in. *Never* try to fit more than one answer into the grid.

If your answer is a whole number such as 2451 or a decimal that only requires four or less slots such as 2.36, then simply enter the number starting at any column. The two examples just written must be started in the first column, but the number 16 can be entered starting in column 1, 2 or 3.

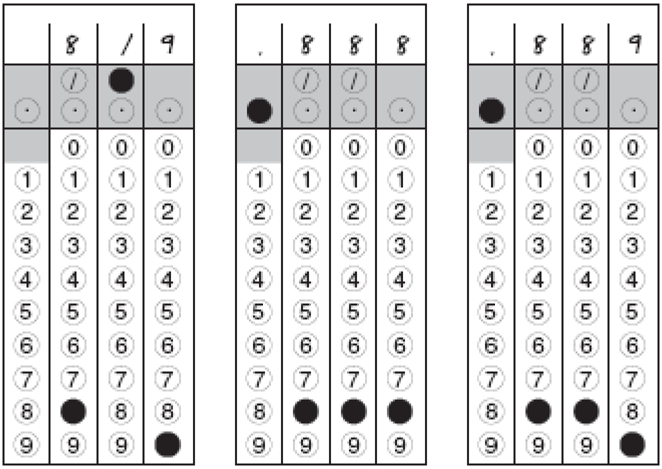
Note that there is no zero in column 1, so if your answer is 0 it must be gridded into column 2, 3 or 4.

Fractions can be gridded in any form as long as there are enough slots. The fraction 2/100 must be reduced to 1/50 simply because the first representation will not fit in the grid.

Fractions can also be converted to decimals before being gridded in. If a decimal cannot fit in the grid, then you can simply *truncate* it to fit. But you must use every slot in this case. For example, the decimal .167777777… can be gridded as .167, but .16 or .17 would both be marked wrong.

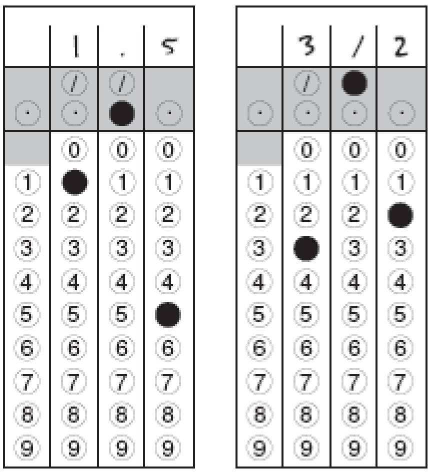
Instead of truncating decimals you can also *round* them. For example, the decimal above could be gridded as .168. Truncating is preferred because there is no thinking involved and you are less likely to make a careless error.

Here are three ways to grid in the number $\frac{8}{9}$.



Never grid-in mixed numerals. If your answer is $2\frac{1}{4}$, and you grid in the mixed numeral $2\frac{1}{4}$, then this will be read as $\frac{21}{4}$ and will be marked wrong. You must either grid in the decimal 2.25 or the improper fraction $\frac{9}{4}$.

Here are two ways to grid in the mixed numeral $1\frac{1}{2}$ correctly.



LESSON 1
HEART OF ALGEBRA

In this lesson we will be reviewing four very basic strategies that can be used to solve a wide range of SAT math problems in all topics and all difficulty levels. Throughout this book you should practice using these four strategies whenever it is possible to do so. You should also try to solve each problem in a more straightforward way.

Start with Choice (B) or (C)

In many SAT math problems you can get the answer simply by trying each of the answer choices until you find the one that works. Unless you have some intuition as to what the correct answer might be, then you should always start in the middle with choice (B) or (C) as your first guess (an exception will be detailed in the next strategy below). The reason for this is simple. Answers are usually given in increasing or decreasing order. So very often if choice (B) or (C) fails you can eliminate one or two of the other choices as well.

Try to answer the following question using this strategy. **Do not** check the solution until you have attempted this question yourself.

LEVEL 2: HEART OF ALGEBRA

$$x - 3 = \sqrt{x + 3}$$

1. What is the solution set of the equation above?

   (A) $\{1\}$
   (B) $\{6\}$
   (C) $\{1,6\}$
   (D) There are no solutions.

See if you can answer this question by starting with choice (B) or (C).

Solution by starting with choice (B): Let's start with choice (B) and guess that the answer is $\{6\}$. We substitute 6 for x into the given equation to get

$$6 - 3 = \sqrt{6 + 3}$$
$$3 = \sqrt{9}$$
$$3 = 3$$

Since this works, we have eliminated choices (A) and (D). But we still need to check to see if 1 works to decide if the answer is (B) or (C).

We substitute 1 for x into the given equation to get

$$1 - 3 = \sqrt{1 + 3}$$
$$-2 = \sqrt{4}$$
$$-2 = 2$$

So 1 is not a solution to the given equation and we can eliminate choice (C). The answer is therefore choice (B).

Important note: Once we see that $x = 6$ is a solution to the given equation, it is **very important** that we make sure there are no answer choices remaining that also contain 6. In this case answer choice (C) also contains 6 as a solution. We therefore must check if 1 is a solution too. In this case it is not.

Solution by starting with choice (C): Let's start with choice (C) and guess that the answer is {1,6}. We begin by substituting 1 for x into the given equation to get the false equation $-2 = 2$ (see the previous solution for details). So 1 is not a solution to the given equation and we can eliminate choice (C). Note that we also eliminate choice (A).

Let's try choice (B) now and guess that the answer is {6}. So we substitute 6 for x into the given equation to get the true equation $3 = 3$ (see the previous solution for details).

Since this works, the answer is in fact choice (B).

Important note: Once we see that $x = 6$ is a solution to the given equation, it is **very important** that we make sure there are no answer choices remaining that also contain 6. In this case we have already eliminated choices (A) and (C), and choice (D) does not contain 6 (in fact choice (D) contains no numbers at all).

Before we go on, try to solve this problem algebraically.

Algebraic solution:

$$x - 3 = \sqrt{x + 3}$$

$$(x - 3)^2 = \left(\sqrt{x + 3}\right)^2$$

$$(x - 3)(x - 3) = x + 3$$

$$x^2 - 6x + 9 = x + 3$$

$$x^2 - 7x + 6 = 0$$

$$(x - 1)(x - 6) = 0$$

$$x - 1 = 0 \text{ or } x - 6 = 0$$

$$x = 1 \text{ or } x = 6$$

When solving algebraic equations with square roots we sometimes generate extraneous solutions. We therefore need to check each of the *potential* solutions 1 and 6 back in the original equation. As we have already seen in the previous solutions 6 <u>is</u> a solution, and 1 <u>is not</u> a solution. So the answer is choice (B).

Notes: (1) Do not worry if you are having trouble understanding all the steps of this solution. We will be reviewing the methods used here later in the book.

(2) Squaring both sides of an equation is not necessarily "reversible." For example, when we square each side of the equation $x = 2$, we get the equation $x^2 = 4$. This new equation has two solutions: $x = 2$ and $x = -2$, whereas the original equation had just one solution: $x = 2$.

This is why we need to check for **extraneous solutions** here.

(3) Solving this problem algebraically is just silly. After finding the potential solutions 1 and 6, we still had to check if they actually worked. But if we had just glanced at the answer choices we would have already known that 1 and 6 were the only numbers we needed to check.

When NOT to Start with Choice (B) or (C)

If the word **least** appears in the problem, then start with the smallest number as your first guess. Similarly, if the word **greatest** appears in the problem, then start with the largest number as your first guess.

Try to answer the following question using this strategy. **Do not** check the solution until you have attempted this question yourself.

LEVEL 2: HEART OF ALGEBRA

2. What is the greatest integer x that satisfies the inequality $2 + \frac{x}{5} < 7$?

   (A) 20
   (B) 22
   (C) 24
   (D) 25

See if you can answer this question by starting with choice (A) or (D).

Solution by plugging in answer choices: Since the word "greatest" appears in the problem, let's start with the largest answer choice, choice (D). Now $2 + \frac{25}{5} = 2 + 5 = 7$ This is just barely too big, and so the answer is choice (C).

Before we go on, try to solve this problem algebraically.

*** Algebraic solution:** Let's solve the inequality. We start by subtracting 2 from each side of the given inequality to get $\frac{x}{5} < 5$. We then multiply each side of this inequality by 5 to get $x < 25$. The greatest integer less than 25 is 24, choice (C).

Take a Guess

Sometimes the answer choices themselves cannot be substituted in for the unknown or unknowns in the problem. But that does not mean that you cannot guess your own numbers. Try to make as reasonable a guess as possible, but do not over think it. Keep trying until you zero in on the correct value.

Try to answer the following question using this strategy. **Do not** check the solution until you have attempted this question yourself.

LEVEL 3: HEART OF ALGEBRA

3. Dana has pennies, nickels and dimes in her pocket. The number of dimes she has is three times the number of nickels, and the number of nickels she has is 2 more than the number of pennies. Which of the following could be the total number of coins in Dana's pocket?

   (A) 15
   (B) 16
   (C) 17
   (D) 18

See if you can answer this question by taking guesses.

*** Solution by taking a guess:** Let's take a guess and say that Dana has 3 pennies. It follows that she has $3 + 2 = 5$ nickels, and $(3)(5) = 15$ dimes. So the total number of coins is $3 + 5 + 15 = 23$. This is too many. So let's guess that Dana has 2 pennies. Then she has $2 + 2 = 4$ nickels, and she has $(3)(4) = 12$ dimes for a total of $2 + 4 + 12 = 18$ coins. Thus, the answer is choice (D).

Before we go on, try to solve this problem the way you might do it in school.

Attempt at an algebraic solution: If we let x represent the number of pennies, then the number of nickels is $x + 2$, and the number of dimes is $3(x + 2)$. Thus, the total number of coins is

$$x + (x + 2) + 3(x + 2) = x + x + 2 + 3x + 6 = 5x + 8.$$

So some possible totals are 13, 18, 23,… which we get by substituting 1, 2, 3,… for x. Substituting 2 in for x gives 18 which is answer choice (D).

Warning: Many students incorrectly interpret "three times the number of nickels" as $3x + 2$. This is not right. The number of nickels is $x + 2$, and so "three times the number of nickels" is $3(x + 2) = 3x + 6$.

Pick a Number

A problem may become much easier to understand and to solve by substituting a specific number in for a variable. Just make sure that you choose a number that satisfies the given conditions.

Here are some guidelines when picking numbers.

(1) Pick a number that is simple but not too simple. In general you might want to avoid picking 0 or 1 (but 2 is usually a good choice).
(2) Try to avoid picking numbers that appear in the problem.
(3) When picking two or more numbers try to make them all different.
(4) Most of the time picking numbers only allows you to eliminate answer choices. So do not just choose the first answer choice that comes out to the correct answer. If multiple answers come out correct you need to pick a new number and start again. But you only have to check the answer choices that have not yet been eliminated.
(5) If there are fractions in the question a good choice might be the least common denominator (lcd) or a multiple of the lcd.
(6) In percent problems choose the number 100.
(7) Do not pick a negative number as a possible answer to a grid-in question. This is a waste of time since you cannot grid a negative number.
(8) If your first attempt does not eliminate 3 of the 4 choices, try to choose a number that's of a different "type." Here are some examples of types:
    (a) A positive integer greater than 1.
    (b) A positive fraction (or decimal) between 0 and 1.
    (c) A negative integer less than -1.
    (d) A negative fraction (or decimal) between -1 and 0.
(9) If you are picking pairs of numbers try different combinations from (8). For example you can try two positive integers greater than 1, two negative integers less than -1, or one positive and one negative integer, etc.

Remember that these are just guidelines and there may be rare occasions where you might break these rules. For example sometimes it is so quick and easy to plug in 0 and/or 1 that you might do this even though only some of the answer choices get eliminated.

Try to answer the following question using this strategy. **Do not** check the solution until you have attempted this question yourself.

LEVEL 3: HEART OF ALGEBRA

$$\frac{x+y}{x}=\frac{2}{9}$$

4. If the equation shown above is true, which of the following must also be true?

(A) $\frac{x}{y}=\frac{9}{11}$

(B) $\frac{x}{y}=-\frac{9}{7}$

(C) $\frac{x-y}{x}=\frac{11}{9}$

(D) $\frac{x-y}{x}=-\frac{9}{7}$

See if you can answer this question by picking numbers.

Solution by picking numbers: Let's choose values for x and y, say $x=9$ and $y=-7$. Notice that we chose these values to make the given equation true.

Now let's check if each answer choice is true or false.

(A) $\frac{9}{-7}=\frac{9}{11}$ False

(B) $\frac{9}{-7}=-\frac{9}{7}$ True

(C) $\frac{9-(-7)}{9}=\frac{11}{9}$ or $\frac{16}{9}=\frac{11}{9}$ False

(D) $\frac{9-(-7)}{9}=-\frac{9}{7}$ or $\frac{16}{9}=-\frac{9}{7}$ False

Since (A), (C), and (D) are each False we can eliminate them. Thus, the answer is choice (B).

Before we go on, try to solve this problem the way you might do it in school.

Algebraic solution 1: $\frac{x+y}{x} = \frac{x}{x} + \frac{y}{x} = 1 + \frac{y}{x}$. So the given equation is equivalent to $1 + \frac{y}{x} = \frac{2}{9}$. Therefore $\frac{y}{x} = \frac{2}{9} - 1 = \frac{2}{9} - \frac{9}{9} = -\frac{7}{9}$, and so $\frac{x}{y} = -\frac{9}{7}$, choice (B).

Note: Most students have no trouble at all adding two fractions with the same denominator. For example,

$$\frac{x}{x} + \frac{y}{x} = \frac{x+y}{x}$$

But these same students have trouble reversing this process.

$$\frac{x+y}{x} = \frac{x}{x} + \frac{y}{x}$$

Note that these two equations are **identical** except that the left and right hand sides have been switched. Note also that to break a fraction into two (or more) pieces, the original denominator is repeated for **each** piece.

Algebraic solution 2: We cross multiply the given equation to get

$$9(x+y) = 2x$$

We now distribute the 9 on the left to get

$$9x + 9y = 2x$$

Now we subtract $2x$ from each side of this last equation to get

$$7x + 9y = 0$$

We subtract $9y$ from each side to get $7x = -9y$.

We can get $\frac{x}{y}$ to one side by performing **cross division.** We do this just like cross multiplication, but we divide instead. Dividing each side of the equation by $7y$ will do the trick (this way we get rid of 7 on the left and y on the right).

$$\frac{x}{y} = \frac{-9}{7} = -\frac{9}{7}$$

This is choice (B).

You're doing great! Let's just practice a bit more. Try to solve each of the following problems by using one of the four strategies you just learned. Then, if possible, solve each problem another way. The answers to these problems, followed by full solutions are at the end of this lesson. **Do not** look at the answers until you have attempted these problems yourself. Please remember to mark off any problems you get wrong.

LEVEL 1: HEART OF ALGEBRA

5. If $3z = \frac{y-5}{2}$ and $z = 5$, what is the value of y ?

   (A) 20
   (B) 25
   (C) 30
   (D) 35

6. If $x > 0$ and $x^4 - 16 = 0$, what is the value of x ?

LEVEL 2: HEART OF ALGEBRA

7. If $\frac{5x}{y} = 10$, what is the value of $\frac{8y}{x}$?

   (A) 4
   (B) 3
   (C) 2
   (D) 1

LEVEL 3: HEART OF ALGEBRA

8. The cost of 5 scarves is d dollars. At this rate, what is the cost, in dollars of 45 scarves?

   (A) $\frac{9d}{5}$

   (B) $\frac{d}{45}$

   (C) $\frac{45}{d}$

   (D) $9d$

9. Bill has cows, pigs and chickens on his farm. The number of chickens he has is four times the number of pigs, and the number of pigs he has is three more than the number of cows. Which of the following could be the total number of these animals?

(A) 28
(B) 27
(C) 26
(D) 25

LEVEL 4: HEART OF ALGEBRA

10. For all real numbers x and y, $|x - y|$ is equivalent to which of the following?

(A) $x + y$
(B) $\sqrt{x - y}$
(C) $(x - y)^2$
(D) $\sqrt{(x - y)^2}$

11. If $k \neq \pm 1$, which of the following is equivalent to $\frac{1}{\frac{1}{k+1}+\frac{1}{k-1}}$.

(A) $2k$
(B) $k^2 - 1$
(C) $\frac{k^2-1}{2k}$
(D) $\frac{2k}{k^2-1}$

12. In the real numbers, what is the solution of the equation $4^{x+2} = 8^{2x-1}$?

(A) $-\frac{7}{4}$
(B) $-\frac{1}{4}$
(C) $\frac{3}{4}$
(D) $\frac{7}{4}$

Answers

1. B	5. D	9. B
2. C	6. 2	10. D
3. D	7. A	11. C
4. B	8. D	12. D

Note: The full solution for question 9 has been omitted because its solution is very similar to the solution for question 3.

Full Solutions

8.

Solution by picking numbers: Let's choose a value for d, say $d = 10$. So 5 scarves cost 10 dollars, and therefore each scarf costs 2 dollars. It follows that 45 scarves cost $(45)(2) = \mathbf{90}$ dollars. **Put a nice big, dark circle around this number so that you can find it easily later.** We now substitute 10 in for d into **all** four answer choices (we use our calculator if we're allowed to).

(A) 90/5 = 18
(B) 10/45
(C) 45/10 = 4.5
(D) 9*10 = 90

Since (D) is the only choice that has become 90, we conclude that (D) is the answer.

Important note: (D) is **not** the correct answer simply because it is equal to 90. It is correct because all 3 of the other choices are **not** 90.

*** Solution using ratios:** We begin by identifying 2 key words. In this case, such a pair of key words is "scarves" and "dollars."

scarves	5	45
dollars	d	x

Notice that we wrote in the number of scarves next to the word scarves, and the cost of the scarves next to the word dollars. Also notice that the cost for 5 scarves is written under the number 5, and the (unknown) cost for 45 scarves is written under the 45. Now draw in the division symbols and equal sign, cross multiply and divide the corresponding ratio to find the unknown quantity x.

$$\frac{5}{d}=\frac{45}{x}$$
$$5x=45d$$
$$x=9d$$

So 45 scarves cost $9d$ dollars, choice (D).

10.

Solution by picking numbers: Let's choose values for x and y, let's say $x=2$ and $y=5$. Then $|x-y|=|2-5|=|-3|=\mathbf{3}$.

Put a nice big dark circle around $\mathbf{3}$ so you can find it easily later. We now substitute $x=2$ and $y=5$ into each answer choice:

(A) 7
(B) $\sqrt{-3}$
(C) $(-3)^2 = 9$
(D) $\sqrt{(-3)^2} = \sqrt{9} = 3$

Since A, B and C each came out incorrect, we can eliminate them. Therefore the answer is choice (D).

*** Solution using the definition of absolute value:** One definition of the absolute value of x is $|x|=\sqrt{x^2}$. So $|x-y|=\sqrt{(x-y)^2}$, choice (D).

Note: Here we have simply replaced x by $x-y$ on both sides of the equation $|x|=\sqrt{x^2}$.

11.

Solution by picking a number: Let's choose a value for k, say $k=2$. Then

$$\frac{1}{\frac{1}{k+1}+\frac{1}{k-1}}=\frac{1}{\frac{1}{2+1}+\frac{1}{2-1}}=\frac{1}{\frac{1}{3}+1}=\frac{1}{\frac{1}{3}+\frac{3}{3}}=\frac{1}{\frac{4}{3}}=\mathbf{\frac{3}{4}}$$

Put a nice big, dark circle around this number so that you can find it easily later. We now substitute 2 in for k into **all** four answer choices (we use our calculator if we're allowed to).

(A) $2*2=4$
(B) $2\char`^2-\ 1=3$
(C) $(2\char`^2-1)/(2*2)=3/4$
(D) $(2*2)/(2\char`^2-1)=4/3$

Since (C) is the only choice that has become $\frac{3}{4}$, we conclude that (C) is the answer.

Important note: (C) is **not** the correct answer simply because it is equal to $\frac{3}{4}$. It is correct because all 3 of the other choices are **not** $\frac{3}{4}$.

***Algebraic solution:** We multiply the numerator and denominator of the complex fraction by $(k+1)(k-1)$ to get

$$\frac{1}{\frac{1}{k+1}+\frac{1}{k-1}} \cdot \frac{(k+1)(k-1)}{(k+1)(k-1)} = \frac{(k+1)(k-1)}{(k-1)+(k+1)} = \frac{k^2-1}{2k}$$

This is choice (C).

Notes: (1) The three simple fractions within this complex fraction are $1 = \frac{1}{1}$, $\frac{1}{k+1}$, and $\frac{1}{k-1}$.

The least common denominator (LCD) of these three fractions is

$$(k+1)(k-1)$$

Note that the least common denominator is just the least common multiple (LCM) of the three denominators. In this problem the LCD is the same as the product of the denominators.

(2) To simplify a complex fraction we multiply each of the numerator and denominator of the fraction by the LCD of all the simple fractions that appear.

(3) Make sure to use the distributive property correctly here.

$$\left(\frac{1}{k+1}+\frac{1}{k-1}\right) \cdot (k+1)(k-1)$$
$$= \left(\frac{1}{k+1}\right) \cdot (k+1)(k-1) + \left(\frac{1}{k-1}\right) \cdot (k+1)(k-1)$$
$$= (k-1)+(k+1)$$

This is how we got the denominator in the second expression in the solution.

(4) Do not worry too much if you are having trouble understanding all the steps of this solution. We will be reviewing the methods used here later in the book.

12.

Solution by starting with choice (C) and using our calculator: Let's start with choice (C) and guess that $x = \frac{3}{4}$. We type in our calculator:

$$4\text{^}(3/4 + 2) \approx 45.255 \quad \text{and} \quad 8\text{^}(2 * 3/4 - 1) \approx 2.828$$

Since these two numbers are different we can eliminate choice (C).

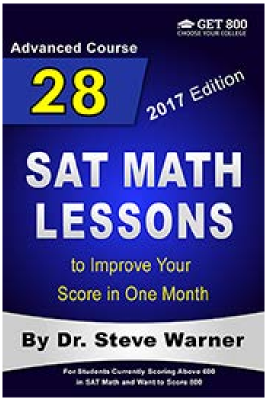
Let's try choice (D) next:

$$4\text{^}(7/4 + 2) \approx 181.019 \quad \text{and} \quad 8\text{^}(2 * 7/4 - 1) \approx 181.019$$

Since they came out the same, the answer is choice (D).

*** Algebraic solution:** The numbers 4 and 8 have a common base of 2. In fact, $4 = 2^2$ and $8 = 2^3$. So we have $4^{x+2} = (2^2)^{x+2} = 2^{2x+4}$ and we have $8^{2x-1} = (2^3)^{2x-1} = 2^{6x-3}$. Thus, $2^{2x+4} = 2^{6x-3}$, and so $2x + 4 = 6x - 3$. We subtract $2x$ from each side of this equation to get $4 = 4x - 3$. We now add 3 to each side of this last equation to get $7 = 4x$. Finally we divide each side of this equation by 4 to get $\frac{7}{4} = x$, choice (D).

Note: For a review of the laws of exponents see lesson 13.



About the Author

Dr. Steve Warner, a New York native, earned his Ph.D. at Rutgers University in Pure Mathematics in May, 2001. While a graduate student, Dr. Warner won the TA Teaching Excellence Award.

After Rutgers, Dr. Warner joined the Penn State Mathematics Department as an Assistant Professor. In September, 2002, Dr. Warner returned to New York to accept an Assistant Professor position at Hofstra University. By September 2007, Dr. Warner had received tenure and was promoted to Associate Professor. He has taught undergraduate and graduate courses in Precalculus, Calculus, Linear Algebra, Differential Equations, Mathematical Logic, Set Theory and Abstract Algebra.

Over that time, Dr. Warner participated in a five year NSF grant, “The MSTP Project,” to study and improve mathematics and science curriculum in poorly performing junior high schools. He also published several articles in scholarly journals, specifically on Mathematical Logic.

Dr. Warner has more than 15 years of experience in general math tutoring and tutoring for standardized tests such as the SAT, ACT and AP Calculus exams. He has tutored students both individually and in group settings.

In February, 2010 Dr. Warner released his first SAT prep book “The 32 Most Effective SAT Math Strategies,” and in 2012 founded Get 800 Test Prep. Since then Dr. Warner has written books for the SAT, ACT, SAT Math Subject Tests and AP Calculus exams.

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