

Prep Official Study Guide Math Companion Solutions to 540 **Math Problems**

By Dr. Steve Warner

SAT Math Problem Explanations For All Tests in the College Board's 2nd Edition Official Study Guide

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SAT Prep Official Study Guide Math Companion

study Guide Math Companion

SAT Math Problem Explanations For All Tests in the College Board's 2nd Edition Official Study Guide

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This book is dedicated to all my students over the past 12 years, I have learned just as much from all of you as you have learned from me.

I would also like to acknowledge Larry Ronaldson and Robert Folatico, thank you for introducing me to the rewarding field of SAT tutoring.

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BLUE BOOK TEST 1 FULLY EXPLAINED SOLUTIONS

SECTION 3

1. Algebra

Solution by plugging in the number: We substitute 4 in for *x* in all five answer choices.

(A) (5)(6) = 30 (B) (5)(3) = 15 (C) (2)(6) = 12 (D) (2)(5) = 10 (E) (0)(8) = 0

We see that 30 is the largest answer so that the answer is choice (A).

Remark: You can minimize the amount of time it takes to do these computations by doing everything in your head and just writing down the final answer. For example, for choice (A) you should add 4 + 1 to get 5, and 4 + 2 to get 6, and then multiply (5)(6) to get 30. Then just write down 30 next to answer choice (A). Repeat this for the rest of the answer choices.

* Quick solution: A quick scan of the first four answer choices will show that choice (A) has factors both of which are at least as large as both factors in each of the next three answer choices. Therefore we can eliminate choices (B), (C), and (D). Choice (E) is easily seen to be 0. So the answer is choice (A).

2. Algebra

* Quick solution: *B* = 7. So *A* = 3(7) = 21. So *C* = 2(21) = 42, choice (E).

Note: In the above solution *A* is an abbreviation for train *A*'s speed in miles per hour, and similarly for *B* and *C*.

Algebraic solution: We are given that A = 3B, C = 2A, and B = 7. Therefore A = 3(7) = 21, and C = 2(21) = 42, choice (E).

3. STATISTICS

Solution by starting with choice (C): We begin by looking at choice (C), and we take a guess that x = 3. Then 5x = 15 and 6x = 18. So, the average is $\frac{3+15+18}{3} = 12$. This is too big. So we can eliminate (C), (D), and (E).

Let's try choice (B) next and guess that x = 2. Then 5x = 10 and 6x = 12. So, the average is $\frac{2+10+12}{3} = 8$. This is correct, so that the answer is choice (B).

For more information on this technique, see **Strategy 1** in *"The 32 Most Effective SAT Math Strategies."*

* Solution by changing averages to sums: We change the average to a sum using the formula

So x + 5x + 6x = (8)(3), or equivalently 12x = 24. Dividing each side of this equation by 12 yields x = 2, choice (B).

Note: The above formula comes from eliminating the denominator in the definition of average: Average = $\frac{Sum}{Number}$

For more information on this technique, see **Strategy 20** in "*The 32 Most Effective SAT Math Strategies.*"

4. FUNCTIONS

* The given statement is equivalent to saying that the graph "passes the vertical line test." So any vertical line should hit the graph at most once. Only choice (D) satisfies this requirement.

5. PERCENTS

* 9 students studied butterflies, and there are 30 students total. Now we have $\frac{9}{30}$ = .3 (use your calculator for this). We change .3 to a percent by multiplying by 100. That is .3(100) = 30%, choice (C).

Some notes on converting between fraction, decimal and percent:

(1) To change a fraction to a decimal, simply perform the division in your calculator, as was done in the solution above.

(2) To change a decimal to a fraction using your TI-84 (or equivalent) calculator simply press the sequence of buttons MATH ENTER ENTER.

(3) To change a fraction to a percent, set up a ratio where the second fraction has denominator 100. Then cross multiply and divide. In this question we could have done this as follows:

$$\frac{9}{30} = \frac{x}{100}$$
$$30x = 900$$
$$x = \frac{900}{30} = 30.$$

(4) To change a decimal to a percent, multiply by 100 as was done above, or equivalently move the decimal point two places to the right (adding zeros if necessary).

(5) To change a percent to a decimal, divide by 100, or equivalently move the decimal point two places to the left (adding zeros if necessary).

(6) To change a percent to a fraction using your TI-84 (or equivalent) calculator simply divide by 100 and press the sequence of buttons MATH ENTER ENTER.

6. GEOMETRY

* The length of \overline{CD} is 10. Since AB = CD, the length of \overline{AB} is also 10. We therefore need to move down 7 to get to *t*, so that *t* = -7, choice (C).

Some details: Since \overline{CD} is a horizontal segment, we compute the length of *CD* by subtracting the *x*-coordinate of *C* from the *x*-coordinate of *D*:

$$CD = 6 - (-4) = 6 + 4 = 10.$$

Similarly, since \overline{AB} is a vertical segment, we compute the length of \overline{AB} by subtracting the *y*-coordinate of *B* from the *y*-coordinate of *A*:

AB = 3 - t.

Since AB = CD, we have 3 - t = 10. So, -t = 7, and t = -7.

7. Algebra

* Let's solve each equation separately.

 $3x^2 = 12$. So $x^2 = 4$. 4y = 12. So y = 3.

Therefore $x^2y = 4(3) = 12$, choice (D).

Note: We solved for x^2 here. There was no need to solve for x since only x^2 was needed.

8. GEOMETRY

The diameters of circles *A*, *B*, and *C* are 4, 8, and 8, respectively. The diameter of the largest circle is then 4 + 8 + 8 = 20. Therefore the radius of the largest circle is $\frac{20}{2} = 10$, choice (D).

Remark: The diameter of a circle is twice the radius, or equivalently, the radius of a circle is half the diameter.

$$d = 2r$$
 or $r = \frac{d}{2}$.

* **Quick solution:** The radius of the largest circle is the sum of the radius of circle A and the diameter of circle C. Note that the diameter of circle C is 2(4) = 8. So we have 2 + 8 = 10, choice (D).

Definitions: The **radius** of a circle is a line segment that joins the center of the circle with any point on its circumference.

The **diameter** of a circle is a line segment connecting the center of the circle with two points on the circumference of the circle.

9. NUMBER THEORY

The total length from 2 to 42 is 42 - 2 = 40. Since there are 5 subintervals between 2 and 40, the length of each of these subintervals is $\frac{40}{5} = 8$. There are 2 subintervals between 2 and *x*, so that

x = 2 + 8 + 8 = 18, choice (D).

* Quick computation: $\frac{42-2}{5} = 8$. So x = 2 + 8 + 8 = 18, choice (D).

Note about intervals and subintervals: If a < b, the length of the interval from a to b is b - a. If there are n subintervals between a and b, the length of each subinterval is $\frac{b-a}{n}$.

10. Geometry

* Since there are 360 degrees in a circle, we have

90 + 30 + 110 + x = 360 230 + x = 360 x = 130, choice (C).

Remarks: (1) 90 comes from the right angle in the picture.

(2) The 70 is not needed in this computation. It is just there to cause confusion.

11. NUMBER THEORY

Let's choose a positive integer whose remainder is 6 when it is divided by 7. A simple way to find such a k is to add 7 and 6. So let k = 13. It follows that k + 2 = 13 + 2 = 15. 7 goes into 15 two times with a remainder of 1, choice (B).

Important: To find a remainder you must perform division **by hand**. Dividing in your calculator does **not** give you a remainder!

Note: A slightly simpler choice for k is k = 6. Indeed, when 6 is divided by 7 we get 0 with 6 left over. Since this choice for k sometimes confuses students I decided to use 7 + 6 = 13 which is the next simplest choice.

Note that in general we can get a value for k by starting with any multiple of 7 and adding 6. So k = 7n + 6 for some integer n.

* Quickest solution: Let k = 6. It follows that k + 2 = 6 + 2 = 8. 7 goes into 8 once with a remainder of 1, choice (B).

Remark: The answer to this problem is independent of our choice for k (assuming that k satisfies the given condition, of course). The method just described does **not** show this. It is not necessary to do so.

For more information on this technique, see **Strategy 4** in "*The 32 Most Effective SAT Math Strategies.*"

For the advanced student: Here is a complete algebraic solution that actually demonstrates the independence of choice for k. The given condition means that we can write k as k = 7n + 6 for some integer n. Then k + 2 = (7n + 6) + 2 = 7n + 8 = 7n + 7 + 1 = 7(n + 1) + 1 = 7z + 1 where z is the integer n + 1. This shows that when k + 2 is divided by 7 the remainder is 1, choice (B).

Calculator Algorithm for computing a remainder: Although performing division in your calculator never produces a remainder, there is a simple algorithm you can perform which mimics long division. Let's find the remainder when 13 is divided by 6 using this algorithm.

Step 1: Perform the division in your calculator: $\frac{13}{6} \sim 2.166667$ Step 2: Multiply the integer part of this answer by the divisor: 2*6 = 12 Step 3: Subtract this result from the dividend to get the remainder:

Definitions: The **integers** are the counting numbers together with their negatives.

The **positive integers** consist of the positive numbers from that set:

12. GEOMETRY

Solution by drawing a picture: Let's plot the first two points.



It should now be clear that the answer is choice (D).

For more information on this technique, see **Strategy 9** in *"The 32 Most Effective SAT Math Strategies."*

* **Quick solution:** A quick glance at the table shows that as depth increases, pressure also increases. This means that the graph has a positive slope. Also, the first row of the table (Depth=0, Pressure=14.7) shows that the *y*-intercept is positive. The only graph that satisfies these two conditions is the graph given by choice (D).

Remark: The second sentence tells us that the pressure increases at a **constant rate** for every foot of descent. This, together with the fact that the entries under "Depth" in the table are equally spaced implies that the graph is a straight line.

* **Complete algebraic solution** (not recommended during the actual SAT): From the first two rows of the table we see that the points (0, 14.7) and (15, 21.375) are on the line. Therefore the slope of the line is $m = \frac{21.375 - 14.7}{15 - 0} = \frac{6.675}{15} = .445$. Also, in the equation y = mx + b, we have that b = 14.7 (because (0,14.7) is the *y*-intercept of the line). So, the equation of the line is y = .445x + 14.7. The only answer choice that possibly matches this equation is choice (D).

For more information on this technique, see **Strategy 28** in "*The 32 Most Effective SAT Math Strategies.*"

Notes:

Slope =
$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

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Lines with positive slope have graphs that go upwards from left to right. Lines with negative slope have graphs that go downwards from left to right. If the slope of a line is zero, it is horizontal. Vertical lines have **no** slope (this is different from zero slope).



no slope

Slope-intercept form of an equation of a line:

y = mx + b

m is the slope of the line and *b* is the *y*-coordinate of the *y*-intercept, ie. the point (0, b) is on the line. Note that this point lies on the *y*-axis.

13. NUMBER THEORY

Let's just compute the first 6 terms of the sequence. The first term is 1. The second term is (-2)(1) = -2. The third term is (-2)(-2) = 4. The fourth term is (-2)(4) = -8. The fifth term is (-2)(-8) = 16. The sixth term is (-2)(16) = -32, choice (E).

* Quick list: If you can keep multiplying by -2 in your head you can form a list very quickly:

So the answer is choice (E).

Remark: The word **product** indicates that a multiplication is to be performed.

14. Algebra

* 2x - 5 and 2x + 5 are **conjugates**. This means that we can multiply them by just multiplying the first terms and last terms and adding these two together. So

$$(2x-5)(2x+5) = 4x^2 - 25$$

So, $4x^2 - 25 = 5$, and therefore $4x^2 = 30$, choice (E).

Remarks:

(1) If *a* and *b* are real numbers, then a + b and a - b are called **conjugates** of each other. We have $(a + b)(a - b) = a^2 - b^2$.

(2) In general, if *a*, *b*, *c*, and *d* are real numbers, we have

(a+b)(c+d) = ac + ad + bc + bd.

The process of expanding the product on the left to get the expression on the right is often called FOILing (here FOIL stands for First, Outer, Inner, Last).

15. GEOMETRY

* First note that the slope of the line is negative so that we can eliminate choices (C), (D), and (E). Now observe that the figure does NOT say "figure not drawn to scale," so we can assume it is. To get from *O* to *B* it is consistent that you need to go right twice as much as you go down. So the slope can be $-\frac{1}{2}$, choice (B).

Note for the advanced student: The condition |p| > |r| is quite confusing and can essentially be ignored in this problem (in the solution above the picture alone was used). For the advanced students |p| = -p (since we move LEFT from the origin to get to point *A*), and |r| = r (since we move UP from the origin to get to point *A*). Therefore the slope of the line is $\frac{|r|}{|p|}$, and since the denominator is larger than the numerator we should move right more than we move down as we draw the line. Don't worry too much if this confuses you. These are just technical details that aren't necessary to solve the problem.

For more information on this technique, see **Strategy 6** in *"The 32 Most Effective SAT Math Strategies."*

See the notes at the end of problem 12 from this section for more information on slope.

16. Algebra

Solution by picking a number: Let's choose a value for b, say b = 2. Then we have

3a + 4(2) = 23a + 8 = 23a = -6a = -2

So 6a + 6b = 6(-2) + 6(2) = 0. Put a nice big, dark circle around 0 so that you can find it easily later. Substituting 2 in for *b* in each answer choice gives the following:

Since (B), (C), (D), and (E) each came out incorrect, the answer is choice (A).

Important note: (A) is **not** the correct answer simply because it is equal to 0. It is correct because all 4 of the other choices are **not** 0. **You absolutely must check all five choices!**

For more information on this technique, see **Strategy 4** in *"The 32 Most Effective SAT Math Strategies.*"

* Algebraic solution: Subtract b from each side of the equation to get

3a + 3b = 0.

Multiplying each side of the equation by 2 yields

$$2(3a + 3b) = 2(0)$$

 $2(3a) + 2(3b) = 0$
 $6a + 6b = 0.$

So the answer is choice (A).

17. GEOMETRY

* Solution using the fact that the figure is drawn to scale: Note that the big triangle is an isosceles right triangle, that is it's a 45, 45, 90 triangle. So $AC = 10\sqrt{2}\sqrt{2} = 10(2) = 20$ (see Remark 2 below). We can assume that the figure is drawn to scale (since it does not say that it isn't drawn to scale). Therefore the length of the shaded rectangle is 10 and the width is 5. So the area of the shaded rectangle is 10(5) = 50, choice (C).

Remarks:

(1) An isosceles right triangle is the same as a 45, 45, 90 right triangle.

(2) We got *AC* by using the formula for a 45, 45, 90 triangle given at the beginning of any math section of the SAT.

(3) The formula for the area of a rectangle is also given at the beginning of any math section of the SAT.

For more information on this technique, see **Strategy 6** in *"The 32 Most Effective SAT Math Strategies."*

Some nitpicky details: For completeness we give the details without assuming that the measurements in the picture are accurate.

We have $AE = CF = 5\sqrt{2}$ by the definition of midpoint. So each small triangle has legs of length 5. It follows that the width of the shaded rectangle is 5, and the length of the shaded rectangle is 20 - 5 - 5 = 10.

Definition: The **midpoint** of a line segment is the point on the segment that divides the segment into two equal parts.

18. FUNCTIONS

* Solution by plugging in points: Let's start with the point $(0,\frac{1}{2})$. Equivalently, $\frac{1}{2} = f(0) = ka^0 = k$.

So the function is now $f(x) = (\frac{1}{2})a^x$. Let's use the point (1,2) to find a. Equivalently, $2 = f(1) = (\frac{1}{2})a^1$ so that a = 4, choice (D). For more information on this technique, see **Strategy 5** in *"The 32 Most Effective SAT Math Strategies."*

19. GEOMETRY

Solution by picking numbers: Let's let e = m = 2. We get the following two pictures.



The picture on the left is the square base of the pyramid. Note that $x = 2\sqrt{2}$. We can see this by observing that the triangle formed from two sides of a square and its diagonal is an isosceles right triangle (or equivalently, a 45, 45, 90 triangle). So we can use the formula for the special triangle or the Pythagorean Theorem. Both of these formulas are given at the beginning of this math section.

The picture on the right consists of the triangle with vertices *V* (the top of the pyramid), the center of the square, and the front right vertex of the square. Note that the side labeled $\frac{x}{2}$ is half the diagonal of the square. It now follows that $\frac{x}{2} = \sqrt{2}$, and we can use the Pythagorean Theorem to find *h*.

$$2^{2} = h^{2} + (\sqrt{2})^{2}$$
$$4 = h^{2} + 2$$
$$h^{2} = 2$$
$$h = \sqrt{2}$$

Put a nice big, dark circle around $\sqrt{2}$ so that you can find it easily later. Substituting 2 in for *m* in each answer choice gives the following:

(A)
$$\frac{2}{\sqrt{2}}$$

(B) $\sqrt{3}$
(C) 2
(D) $\frac{4}{\sqrt{3}}$
(E) $2\sqrt{2}$

Uh oh! It appears as if none of the answers are correct. But actually, choice (A) is correct because $\frac{2}{\sqrt{2}} = \sqrt{2}$. You can see this by putting each of these numbers in your calculator and observing that you get the same decimal approximation. Alternatively, you can rationalize the denominator of $\frac{2}{\sqrt{2}}$ by multiplying each of the numerator and denominator by $\sqrt{2}$ as follows.

$$\left(\frac{2}{\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{2\sqrt{2}}{2} = \sqrt{2}.$$

Since (B), (C), (D), and (E) each came out incorrect, the answer is choice (A).

Important note: (A) is **not** the correct answer simply because it is equal to $\sqrt{2}$. It is correct because all 4 of the other choices are **not** $\sqrt{2}$. You absolutely must check all five choices!

For more information on this technique, see **Strategy 4** in *"The 32 Most Effective SAT Math Strategies."*

* Algebraic solution: Let's draw two pictures as in the last solution.



A diagonal of the square base has length $x = m\sqrt{2}$. So the length from the center to the lower right corner of the square is $\frac{x}{2} = \frac{m\sqrt{2}}{2}$.

Now notice that we have a right triangle whose legs are labeled by h and $\frac{x}{2} = \frac{m\sqrt{2}}{2}$, and whose hypotenuse is labeled by e = m. By the Pythagorean Theorem, $m^2 = h^2 + (\frac{m\sqrt{2}}{2})^2$. Now, let's square the rightmost term to get $m^2 = h^2 + \frac{m^2 \cdot 2}{4}$, or equivalently

$$m^2 = h^2 + \frac{m^2}{2}$$

Now multiply each term by 2 (to eliminate the denominator).

$$2m^2 = 2h^2 + m^2$$
. So $m^2 = 2h^2$, and $h^2 = \frac{m^2}{2}$. Therefore $h = \frac{m}{\sqrt{2}}$, choice (A).

20. NUMBER THEORY

Solution by picking numbers: Let's pick a number for k, say k = 10, so that the salesperson's commission is 10% of the selling price of the car. The total sale price for the two cars is 2(14,000) = \$28,000, so that the commission is (28,000)(.1) = 2800. Put a nice big, dark circle around this number.

Now let's substitute 10 in for *k* in each answer choice.

Since (B), (C), (D), and (E) are all incorrect, the answer is choice (A).

Important note: (A) is **not** the correct answer simply because it is equal to 2800. It is correct because all four of the other choices are **not** 2800.

Remark: This is a percent problem, so an even nicer choice for k would be k = 100. Try solving the problem again with this choice for k.

For more information on this technique, see **Strategy 4** in "*The 32 Most Effective SAT Math Strategies.*"

* Algebraic solution: The total sale price for the two cars is 2(14,000) = \$28,000, so that the commission is $(28,000)(\frac{k}{100}) = 280k$, choice (A).

About the Author

Steve Warner, a New York native, earned his Ph.D. at Rutgers University in Pure



Mathematics in May, 2001. While a graduate student, Dr. Warner won the TA Teaching Excellence Award.

After Rutgers, Dr. Warner joined the Penn State Mathematics Department as an Assistant Professor. In September, 2002, Dr. Warner returned to New York to accept an Assistant Professor position at Hofstra University. By September 2007, Dr. Warner had received tenure and was promoted to Associate Professor. He has taught undergraduate and graduate courses in Precalculus, Calculus, Linear Algebra,

Differential Equations, Mathematical Logic, Set Theory and Abstract Algebra.

Over that time, Dr. Warner participated in a five year NSF grant, "The MSTP Project," to study and improve mathematics and science curriculum in poorly performing junior high schools. He also published several articles in scholarly journals, specifically on Mathematical Logic.

Dr. Warner has over 15 years of experience in general math tutoring and over 10 years of experience in SAT math tutoring. He has tutored students both individually and in group settings.

In February, 2010 Dr. Warner released his first SAT prep book "The 32 Most Effective SAT Math Strategies." The second edition of this book was released in January, 2011. In February, 2012 Dr. Warner released his second SAT prep book "320 SAT Math Problems arranged by Topic and Difficulty Level." Between September 2012 and January 2013 Dr. Warner released his three book series "28 SAT Math Lessons to Improve Your Score in One Month."

Currently Dr. Warner lives in Staten Island with his two cats, Achilles and Odin. Since the age of 4, Dr. Warner has enjoyed playing the piano—especially compositions of Chopin as well as writing his own music. He also maintains his physical fitness through weightlifting.

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Teachers and tutors may have their own personal philosophies, but there is one thing that they all agree on - every student should be attempting the questions from the practice SATs given in *The Official SAT Study Guide*.

The Complete Official SAT Study Guide Companion contains solutions to every math question from each of the ten SATs in the 2nd edition of the Official SAT Study Guide.

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Using this book you will learn to solve the SAT math problems from the Official SAT Study Guide in clever and efficient ways that will have you spending less time on each problem, and answering even the most difficult of these questions with ease. All definitions and concepts are reviewed in detail as they come up with each question.

Here's to your success on the SAT, in college, and in life.



Steve Warner earned his Ph.D. at Rutgers University in May, 2001, and he currently works as an Associate Professor of Mathematics at Hofsra University. He has over 15 years of experience in general math tutoring and over 10 years of experience in SAT math tutoring