

Intermediate Course

28

2017 Edition

SAT MATH LESSONS

to Improve Your
Score in One Month

By Dr. Steve Warner

For Students Currently Scoring Between
500 and 600 in SAT Math

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I N T R O D U C T I O N

STUDYING FOR SUCCESS

S

his book was written specifically for the student currently scoring between a 500 and 600 in SAT math. Results will vary, but if you are such a student and you work through the lessons in this book, then you will see a substantial improvement in your score.

This book has been cleverly designed to enforce the study habits that I constantly find students ignoring despite my repeated emphasis on how important they are. Many students will learn and understand the strategies I teach them, but this is not enough. This book will force the student to internalize these strategies so that the appropriate strategy is actually used when it is needed. Most students will attempt the problems that I suggest that they work on, but again, this is not enough. All too often students dismiss errors as “careless” and neglect to redo problems they have answered incorrectly. This book will minimize the effect of this neglect.

The book you are now reading is self-contained. Each lesson was carefully created to ensure that you are making the most effective use of your time while preparing for the SAT. The initial lessons are quite focused ensuring that the reader learns and practices one strategy and one topic at a time. In the beginning the focus is on Level 1, 2 and 3 problems, and little by little Level 4 problems will be added into the mix. It should be noted that a score of 700 can usually be attained without ever attempting a Level 5 problem. That said, some Level 5 problems will appear late in the book for those students that show accelerated improvement. The reader of this book should not feel obligated to work on these harder problems the first time they go through this book.

There are two math sections on the SAT: one where a calculator is allowed and one where it is not. I therefore recommend trying to solve as many problems as possible both with and without a calculator. If a calculator is required for a specific problem it will be marked with an asterisk (*).

1. Using this book effectively

- Begin studying at least three months before the SAT.
- Practice SAT math problems ten to twenty minutes each day.
- Choose a consistent study time and location.

You will retain much more of what you study if you study in short bursts rather than if you try to tackle everything at once. So try to choose about a twenty minute block of time that you will dedicate to SAT math each day. Make it a habit. The results are well worth this small time commitment. Some students will be able to complete each lesson within this ten to twenty minute block of time. Others may take a bit longer. If it takes you longer than twenty minutes to complete a lesson, you have two options. You can stop when twenty minutes are up and then complete the lesson the following day, or you can finish the lesson and then take a day off from SAT prep that week.

- Every time you get a question wrong, **mark it off, no matter what your mistake.**
- Begin each lesson by first redoing the problems from previous lessons on the same topic that you have marked off.
- If you get a problem wrong again, **keep it marked off.**

As an example, before you begin the third “Heart of Algebra” lesson (Lesson 9), you should redo all the problems you have marked off from the first two “Heart of Algebra” lessons (Lessons 1 and 5). Any question that you get right you can “unmark” while leaving questions that you get wrong marked off for the next time. If this takes you the full twenty minutes, that is okay. Just begin the new lesson the next day.

Note that this book often emphasizes solving each problem in more than one way. Please listen to this advice. The same question is never repeated on any SAT (with the exception of questions from the experimental sections) so the important thing is learning as many techniques as possible. Being able to solve any specific problem is of minimal importance. The more ways you have to solve a single problem the more prepared you will be to tackle a problem you have never seen before, and the quicker you will be able to solve that problem. Also, if you have multiple methods for solving a single problem, then on the actual SAT when you “check over” your work you will be able to redo each problem in a different way. This will eliminate all “careless” errors on the actual exam. Note that in this book the quickest solution to any problem will always be marked with an asterisk (*).

2. Calculator use.

- Use a TI-84 or comparable calculator if possible when practicing and during the SAT.
- Make sure that your calculator has fresh batteries on test day.
- You may have to switch between DEGREE and RADIAN modes during the test. If you are using a TI-84 (or equivalent) calculator press the MODE button and scroll down to the third line when necessary to switch between modes.

Below are the most important things you should practice on your graphing calculator.

- Practice entering complicated computations in a single step.
- Know when to insert parentheses:
 - Around numerators of fractions
 - Around denominators of fractions
 - Around exponents
 - Whenever you actually see parentheses in the expression

Examples:

We will substitute a 5 in for x in each of the following examples.

<u>Expression</u>	<u>Calculator computation</u>
$\frac{7x+3}{2x-11}$	$(7*5 + 3)/(2*5 - 11)$
$(3x-8)^{2x-9}$	$(3*5 - 8)^(2*5 - 9)$

- Clear the screen before using it in a new problem. The big screen allows you to check over your computations easily.
- Press the **ANS** button (**2ND (-)**) to use your last answer in the next computation.
- Press **2ND ENTER** to bring up your last computation for editing. This is especially useful when you are plugging in answer choices, or guessing and checking.
- You can press **2ND ENTER** over and over again to cycle backwards through all the computations you have ever done.
- Know where the $\sqrt{\quad}$, π , and \wedge buttons are so you can reach them quickly.
- Change a decimal to a fraction by pressing **MATH ENTER ENTER**.
- Press the **MATH** button - in the first menu that appears you can take cube roots and n th roots for any n . Scroll right to **NUM** and you have **lcm()** and **gcd()**.
- Know how to use the **SIN**, **COS** and **TAN** buttons as well as **SIN⁻¹**, **COS⁻¹** and **TAN⁻¹**.

You may find the following graphing tools useful.

- Press the **Y=** button to enter a function, and then hit **ZOOM 6** to graph it in a standard window.
- Practice using the **WINDOW** button to adjust the viewing window of your graph.
- Practice using the **TRACE** button to move along the graph and look at some of the points plotted.
- Pressing **2ND TRACE** (which is really **CALC**) will bring up a menu of useful items. For example selecting **ZERO** will tell you where the graph hits the x -axis, or equivalently where the function is zero. Selecting **MINIMUM** or **MAXIMUM** can find the vertex of a parabola. Selecting **INTERSECT** will find the point of intersection of 2 graphs.

3. Tips for taking the SAT

Each of the following tips should be used whenever you take a practice SAT as well as on the actual exam.

Check your answers properly: When you go back to check your earlier answers for careless errors *do not* simply look over your work to try to catch a mistake. This is usually a waste of time.

- When “checking over” problems you have already done, **always redo the problem from the beginning** without looking at your earlier work.
- If possible use a different method than you used the first time.

For example, if you solved the problem by picking numbers the first time, try to solve it algebraically the second time, or at the very least pick different numbers. If you do not know, or are not comfortable with a different method, then use the same method, but do the problem from the beginning and do not look at your original solution. If your two answers do not match up, then you know that this is a problem you need to spend a little more time on to figure out where your error is.

This may seem time consuming, but that is okay. It is better to spend more time checking over a few problems, than to rush through a lot of problems and repeat the same mistakes.

Take a guess whenever you cannot solve a problem: There is no guessing penalty on the SAT. Whenever you do not know how to solve a problem take a guess. Ideally you should eliminate as many answer choices as possible before taking your guess, but if you have no idea whatsoever do not waste time overthinking. Simply put down an answer and move on. You should certainly mark it off and come back to it later if you have time.

Pace yourself: Do not waste your time on a question that is too hard or will take too long. After you have been working on a question for about 1 minute you need to make a decision. If you understand the question and think that you can get the answer in another 30 seconds or so, continue to work on the problem. If you still do not know how to do the problem or you are using a technique that is going to take a long time, mark it off and come back to it later if you have time.

Feel free to take a guess. But you still want to leave open the possibility of coming back to it later. Remember that every problem is worth the same amount. Do not sacrifice problems that you may be able to do by getting hung up on a problem that is too hard for you.

Attempt the right number of questions: There are two math sections on the SAT – one where a calculator is allowed and one where a calculator is not allowed. The calculator section has 30 multiple choice (mc) questions and 8 free response (grid in) questions. The non-calculator section has 15 multiple choice (mc) questions and 5 free response (grid in) questions.

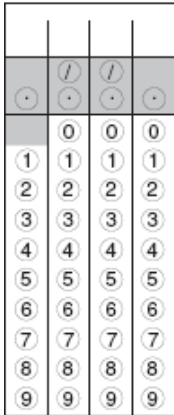
You should first make sure that you know what you got on your last SAT practice test, actual SAT, or actual PSAT (whichever you took last). What follows is a general goal you should go for when taking the exam.

Score	MC (Calculator Allowed)	Grid In (Calculator Allowed)	MC (Calculator Not Allowed)	Grid In (Calculator Not Allowed)
< 330	10/30	3/8	4/15	1/5
330 – 370	15/30	4/8	6/15	2/5
380 – 430	18/30	5/8	8/15	2/5
440 – 490	21/30	6/8	9/15	3/5
500 – 550	24/30	6/8	11/15	4/5
560 – 620	27/30	7/8	13/15	4/5
630 – 800	30/30	8/8	15/15	5/5

For example, a student with a current score of 530 should attempt 24 multiple choice questions and 6 grid ins from the section where a calculator is allowed, and 11 multiple choice questions and 4 grid in questions from the section where a calculator is not allowed.

This is *just* a general guideline. Of course it can be fine-tuned. As a simple example, if you are particularly strong at Algebra problems, but very weak at Geometry and Trig problems, then you may want to try every Algebra problem no matter where it appears, and you may want to reduce the number of Geometry and Trig problems you attempt.

Grid your answers correctly: The computer only grades what you have marked in the bubbles. The space above the bubbles is just for your convenience, and to help you do your bubbling correctly.



Never mark more than one circle in a column or the problem will automatically be marked wrong. You do not need to use all four columns. If you do not use a column just leave it blank.

The symbols that you can grid in are the digits 0 through 9, a decimal point, and a division symbol for fractions. Note that there is no negative symbol. So answers to grid-ins *cannot* be negative. Also, there are only four slots, so you cannot get an answer such as 52,326.

Sometimes there is more than one correct answer to a grid-in question. Simply choose one of them to grid-in. *Never* try to fit more than one answer into the grid.

If your answer is a whole number such as 2451 or a decimal that only requires four or less slots such as 2.36, then simply enter the number starting at any column. The two examples just written must be started in the first column, but the number 16 can be entered starting in column 1, 2 or 3.

Note that there is no zero in column 1, so if your answer is 0 it must be gridded into column 2, 3 or 4.

Fractions can be gridded in any form as long as there are enough slots. The fraction $2/100$ must be reduced to $1/50$ simply because the first representation will not fit in the grid.

Fractions can also be converted to decimals before being gridded in. If a decimal cannot fit in the grid, then you can simply *truncate* it to fit. But you must use every slot in this case. For example, the decimal .16777777... can be gridded as .167, but .16 or .17 would both be marked wrong.

Instead of truncating decimals you can also *round* them. For example, the decimal above could be gridded as .168. Truncating is preferred because there is no thinking involved and you are less likely to make a careless error.

Here are three ways to grid in the number $\frac{8}{9}$.

	8	/	9
·	/	●	·
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	●	8	8
9	9	9	●

.	8	8	8
●	/	/	·
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	●	●	●
9	9	9	9

.	8	8	9
●	/	/	·
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	●	●	8
9	9	9	●

Never grid-in mixed numerals. If your answer is $2\frac{1}{4}$, and you grid in the mixed numeral $2\frac{1}{4}$, then this will be read as $\frac{21}{4}$ and will be marked wrong. You must either grid in the decimal 2.25 or the improper fraction $\frac{9}{4}$.

Here are two ways to grid in the mixed numeral $1\frac{1}{2}$ correctly.

	1	.	5
·	/	/	·
0	0	0	0
1	●	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	●
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

	3	/	2
·	/	●	·
0	0	0	0
1	1	1	1
2	2	2	●
3	●	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

LESSON 1

HEART OF ALGEBRA

Start with Choice (B) or (C)

In many SAT math problems you can get the answer simply by trying each of the answer choices until you find the one that works. Unless you have some intuition as to what the correct answer might be, then you should always start in the middle with choice (B) or (C) as your first guess (an exception will be detailed in the next strategy below). The reason for this is simple. Answers are usually given in increasing or decreasing order. So very often if choice (B) or (C) fails you can eliminate one or two of the other choices as well.

Try to answer the following question using this strategy. **Do not** check the solution until you have attempted this question yourself.

LEVEL 1: HEART OF ALGEBRA

1. If $7 + x + x = 3 + x + x + x$, what is the value of x ?

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Solution by starting with choice (C): We start with choice (C) and substitute 3 in for x on each side of the equation.

$$7 + 3 + 3 = 3 + 3 + 3 + 3$$

$$13 = 12$$

Since this is false, we can eliminate choice (C). A little thought should allow you to eliminate choices (A) and (B) as well (don't worry if you don't see this – just take another guess). Let's try choice (D) next.

$$7 + 4 + 4 = 3 + 4 + 4 + 4$$

$$15 = 15$$

Thus, the answer is choice (D).

Before we go on, try to solve this problem in two other ways.

(1) Algebraically (the way you would do it in school).

(2) By “striking off x ’s.”

Here is a hint for method (2):

Hint: If the same expression appears as a **term** on each side of an equation, you can simply cross out each of these expressions, and the equation remains “balanced.”

Algebraic solution: Here is a quick algebraic solution to the problem.

$$\begin{aligned}7 + x + x &= 3 + x + x + x \\7 + 2x &= 3 + 3x \\7 &= 3 + x \\4 &= x\end{aligned}$$

Thus, the answer is choice (D).

Remark: We can begin with an algebraic solution, and then switch to the easier method. For example, we can write $7 + 2x = 3 + 3x$, and then start substituting in the answer choices from here. This will take less time than the first method, but more time than the algebraic method.

* **(2) Striking off x ’s:** When the same term appears on each side of an equation we can simply delete that term from both sides. In this problem we can strike off two x ’s from each side to get

$$7 = 3 + x.$$

This becomes $4 = x$, choice (D).

When NOT to Start with Choice (B) or (C)

If the word **least** appears in the problem, then start with the smallest number as your first guess. Similarly, if the word **greatest** appears in the problem, then start with the largest number as your first guess.

Try to answer the following question using this strategy. **Do not** check the solution until you have attempted this question yourself.

LEVEL 1: HEART OF ALGEBRA

2. For which of the following values of k will the value of $11k - 12$ be greater than 21?
- (A) 1
 - (B) 2
 - (C) 3
 - (D) 4

*** Solution by starting with choice (D):** Since the word **greater** appears in the problem let's start with the largest number for our first guess. This is choice (D).

$$11k - 12 = 11 \cdot 4 - 12 = 44 - 12 = 32.$$

Since 32 is greater than 21, the answer is choice (D).

Before we go on, try to solve this problem algebraically (without using the answer choices).

Algebraic solution:

$$\begin{aligned}11k - 12 &> 21 \\11k &> 33 \\k &> 3\end{aligned}$$

The only answer choice with a number greater than 3 is choice (D).

You're doing great! Let's just practice a bit more. Try to solve each of the following problems by using one of the two strategies you just learned. Then, if possible, solve each problem another way. The answers to these problems, followed by full solutions are at the end of this lesson. **Do not** look at the answers until you have attempted these problems yourself. Please remember to mark off any problems you get wrong.

LEVEL 1: HEART OF ALGEBRA

3. If $5(x - 7) = 4(x - 8)$, what is the value of x ?
- (A) 1
 - (B) 2
 - (C) 3
 - (D) 4

4. If $4^2 = 2^z$, then $z =$
- (A) 4
 - (B) 3
 - (C) 2
 - (D) 1
5. If $3c + 2 < 11$, which of the following CANNOT be the value of c ?
- (A) 0
 - (B) 1
 - (C) 2
 - (D) 3

$$\frac{5 + \Delta}{2} = 8\frac{1}{2}$$

6. What number, when used in place of Δ above, makes the statement true?
- (A) 4
 - (B) 5
 - (C) 9
 - (D) 12

LEVEL 2: HEART OF ALGEBRA

7. * If $6^{x+1} = 7776$, what is the value of x ?
- (A) 6
 - (B) 5
 - (C) 4
 - (D) 3

LEVEL 3: HEART OF ALGEBRA

8. There is the same number of cows, pigs and chickens being transported to a farm. When the transport arrives at the farm, 4 cows are taken off the truck and 8 chickens are placed on the truck. If there are now twice as many pigs as cows on the truck, and twice as many chickens as pigs on the truck, how many chickens are on the truck?

- (A) 6
 (B) 8
 (C) 12
 (D) 16

Definitions Used in This Lesson

$x < y$ means “ x is less than y .”

For example, $2 < 3$ and $-4 < 0$ are TRUE, whereas $6 < 5$ is FALSE.

$x > y$ means “ x is greater than y .”

For example, $3 > 2$ and $0 > -4$ are TRUE, whereas $5 > 6$ is FALSE.

It sometimes helps to remember that for $<$ and $>$, the symbol always points to the smaller number.

Answers

- | | |
|------|------|
| 1. D | 5. D |
| 2. D | 6. D |
| 3. C | 7. C |
| 4. A | 8. D |

Full Solutions

3.

Solution by starting with choice (C): We start with choice (C) and substitute 3 in for x in the given equation.

$$\begin{aligned} 5(x - 7) &= 4(x - 8) \\ 5(3 - 7) &= 4(3 - 8) \\ 5(-4) &= 4(-5) \\ -20 &= -20 \end{aligned}$$

Thus, the answer is choice (C).

* **Algebraic solution:**

$$\begin{aligned} 5(x - 7) &= 4(x - 8) \\ 5x - 35 &= 4x - 32 \\ x &= 3 \end{aligned}$$

Thus, the answer is choice (C).

Note: To get from the first to the second equation we used the distributive property on each side of the equation. This property will be covered in detail in Lesson 7.

4.

Solution by starting with choice (C): First note that $4^2 = 16$. Now let's begin with choice (C). We substitute 2 in for z to get that $2^z = 2^2 = 4$. This is too small so we can eliminate choices (C) and (D). We next try choice (B). We substitute 3 in for z to get $2^z = 2^3 = 8$. This is still too small so we can eliminate choice (B). The answer must therefore be (A). We should still check that it works. We substitute 4 in for z and we get $2^z = 2^4 = 16$. So the answer is indeed choice (A).

* **Direct solution:** $4^2 = 16 = 2^4$. So $z = 4$. Thus, the answer is (A).

5.

Solution by starting with choice (D): We start with choice (D) and substitute 3 in for c in the given inequality.

$$\begin{aligned} 3c + 2 &< 11 \\ 3(3) + 2 &< 11 \\ 9 + 2 &< 11 \\ 11 &< 11 \end{aligned}$$

Since this is FALSE, the answer is choice (D).

* **Remark:** This is actually a slight variation of the second strategy. A moment's thought should tell you that we are looking for a number that is too big. So the largest number given must be the answer.

Algebraic solution:

$$\begin{aligned} 3c + 2 &< 11 \\ 3c &< 9 \\ c &< 3 \end{aligned}$$

Thus, the answer is choice (D).

6.

Solution by starting with choice (C): We start with choice (C) and substitute 9 in for Δ in the given equation.

$$\begin{aligned}\frac{5+9}{2} &= 8\frac{1}{2} \\ \frac{14}{2} &= 8\frac{1}{2} \\ 7 &= 8\frac{1}{2}\end{aligned}$$

The equation is false. So we can eliminate choices (A), (B), and (C). So the answer must be choice (D). Let's verify this. We substitute 12 for Δ .

$$\begin{aligned}\frac{5+12}{2} &= 8\frac{1}{2} \\ \frac{17}{2} &= 8\frac{1}{2} \\ 8\frac{1}{2} &= 8\frac{1}{2}\end{aligned}$$

So yes, the answer is choice (D).

*** Algebraic solution:**

$$\begin{aligned}\frac{5+\Delta}{2} &= 8\frac{1}{2} \\ 5 + \Delta &= 17 \\ \Delta &= 12\end{aligned}$$

This is choice (D).

7.

Solution by starting with choice (C): We start with choice (C) and substitute 4 in for x in the given equation. We type in our calculator $6^{(4+1)} = 7776$. Thus, the answer is choice (C).

Calculator note: Instead of typing $6^{(4+1)}$ in our calculator, we can add 4 and 1 in our head (to get 5), and type 6^5 instead.

*** Algebraic solution:** We rewrite the equation so that each side has the same base (in this case the common base is 6). $6^{x+1} = 6^5$. Now that the bases are the same, so are the exponents. Thus, $x+1 = 5$, and therefore $x = 4$, choice (C).

8.

Solution by starting with choice (B): If there are 8 chickens, then there are 4 pigs, and 2 cows. That means there were originally 0 chickens, 4 pigs, and 6 cows. Since these numbers are not equal we can eliminate choice (B), and choice (A) as well.

Let's try choice (C) next. If there are 12 chickens, then there are 6 pigs, and 3 cows. That means there were originally 4 chickens, 6 pigs, and 7 cows. Again, these numbers are not equal so we can eliminate (C).

Let's verify that the answer is choice (D). If there are 16 chickens, then there are 8 pigs, and 4 cows. That means there were originally 8 of each. So the answer is choice (D).

* **Algebraic solution:** Let x be the original number of chickens (so x is also the original number of pigs, and the original number of cows). We then have

$$x = 2(x - 4) \quad \text{and} \quad x + 8 = 2x$$

Each of these equations has the unique solution $x = 8$. So the number of chickens is

$$x + 8 = 8 + 8 = 16, \text{ choice (D).}$$

Caution: Before choosing your answer always double check what the question is asking for. In this case we must find the number of chickens which is $x + 8$, **not** x .

Detailed formal solutions of the above two equations:

$$\begin{array}{rcl} x = 2(x - 4) & & x + 8 = 2x \\ x = 2x - 8 & & 8 = x \\ -x = -8 & & \\ x = 8 & & \end{array}$$

OPTIONAL MATERIAL

Informal and Formal Algebra

Suppose we are asked to solve for x in the following equation:

$$x + 3 = 8$$

In other words, we are being asked for a number such that when we add 3 to that number we get 8. It is not too hard to see that $5 + 3 = 8$, so that $x = 5$.

I call the technique above solving this equation **informally**. In other words, when we solve algebraic equations informally we are solving for the variable very quickly in our heads. I sometimes call this performing “**mental math.**”

We can also solve for x **formally** by subtracting 3 from each side of the equation:

$$\begin{array}{r} x + 3 = 8 \\ -3 \quad -3 \\ \hline x \quad = 5 \end{array}$$

In other words, when we solve an algebraic equation formally we are writing out all the steps – just as we would do it on a test in school.

To save time on the SAT you should practice solving equations informally as much as possible. And you should also practice solving equations formally – this will increase your mathematical skill level.

Let’s try another:

$$5x = 30$$

Informally, 5 times 6 is 30, so we see that $x = 6$.

Formally, we can divide each side of the equation by 5:

$$\begin{array}{r} 5x = 30 \\ \underline{5 \quad 5} \\ x = 6 \end{array}$$

Now let’s get a little harder:

$$5x + 3 = 48$$

We can still do this informally. First let's figure out what number plus 3 is 48. Well, 45 plus 3 is 48. So $5x$ is 45. So x must be 9.

Here is the formal solution:

$$\begin{array}{r} 5x + 3 = 48 \\ \underline{-3 \quad -3} \\ 5x = 45 \\ \underline{5 \quad 5} \\ x = 9 \end{array}$$

Now practice some on your own. Try to solve each of the following equations for x both informally, and formally. The answers are below:

1. $x + 17 = 20$

2. $6x = 24$

3. $\frac{x}{12} = 2$

4. $7x - 4 = 24$

5. $\frac{2x-3}{5} = 2$

6. $5(x - 7) = 40$

7. $2^x = 8$

8. $\frac{5+x}{2} = 8\frac{1}{2}$

9. $5^{x+1} = 125$

10. $3^x + 4 = 31$

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Answers

1. 3

2. 4

3. 24

4. 4

5. $13/2$ or 6.5

6. 15

7. 3

8. 12

9. 2

10. 3



About the Author

Dr. Steve Warner, a New York native, earned his Ph.D. at Rutgers University in Pure Mathematics in May, 2001. While a graduate student, Dr. Warner won the TA Teaching Excellence Award.



After Rutgers, Dr. Warner joined the Penn State Mathematics Department as an Assistant Professor. In September, 2002, Dr. Warner returned to New York to accept an Assistant Professor position at Hofstra University. By September 2007, Dr. Warner had received tenure and was promoted to Associate Professor. He has taught undergraduate and graduate courses in Precalculus, Calculus, Linear Algebra, Differential Equations, Mathematical Logic,

Set Theory and Abstract Algebra.

Over that time, Dr. Warner participated in a five year NSF grant, “The MSTP Project,” to study and improve mathematics and science curriculum in poorly performing junior high schools. He also published several articles in scholarly journals, specifically on Mathematical Logic.

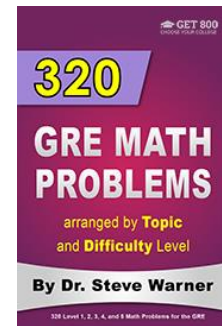
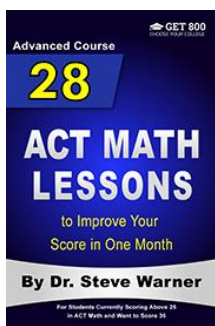
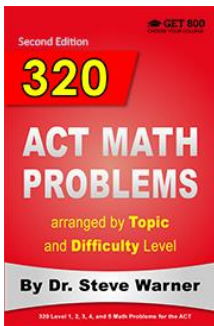
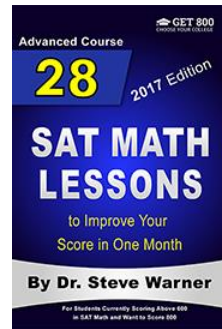
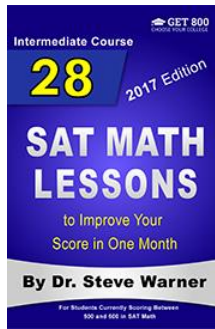
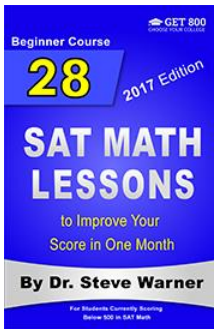
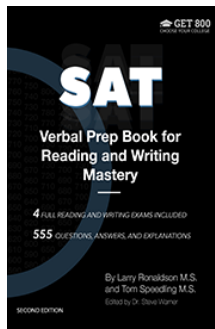
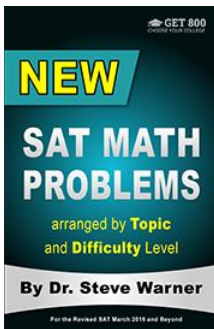
Dr. Warner has more than 15 years of experience in general math tutoring and tutoring for standardized tests such as the SAT, ACT and AP Calculus exams. He has tutored students both individually and in group settings.

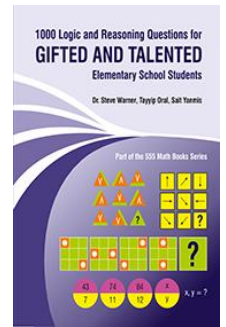
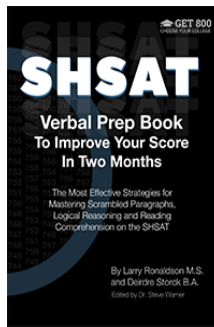
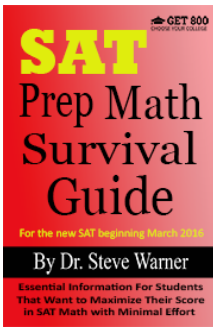
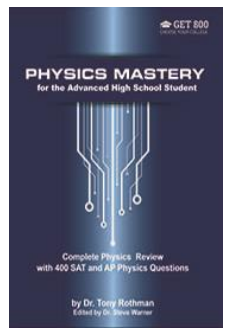
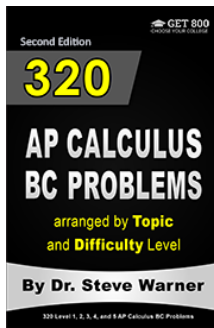
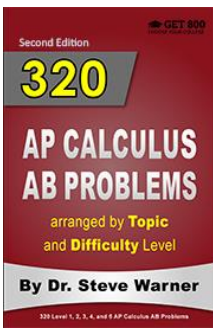
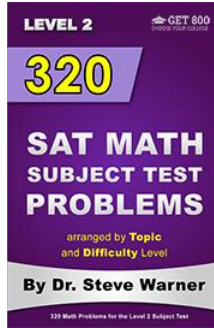
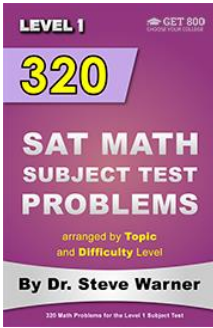
In February, 2010 Dr. Warner released his first SAT prep book “The 32 Most Effective SAT Math Strategies,” and in 2012 founded Get 800 Test Prep. Since then Dr. Warner has written books for the SAT, ACT, SAT Math Subject Tests and AP Calculus exams.

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