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THE SCHOLARLY UNICORN'S SAT MATH ADVANCED GUIDE

DR. STEVE WARNER

WITH 1000 PROBLEMS & 48 LESSONS

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The Scholarly Unicorn's SAT Math Advanced Guide with 1000 Problems and 48 Lessons

Self-Preparation Manual

Dr. Steve Warner

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Solving Linear Equations

Must We Always Be So Formal?

Suppose we are asked to solve for x in the following equation:

$$x - 2 = 7$$

In other words, we are being asked for a number such that when we subtract 2 from that number we get 7. It is not too hard to see that $9 - 2 = 7$, so that $x = 9$.

I call the technique above solving this equation ***informally***. In other words, when we solve algebraic equations informally we are solving for the variable very quickly in our heads.

We can also solve for x ***formally*** by adding 2 to each side of the equation:

$$\begin{array}{rcl} x - 2 & = & 7 \\ \underline{\quad +2} & & \underline{+\,2} \\ x & = & 9 \end{array}$$

In other words, when we solve an algebraic equation formally we are writing out all the steps – just as we would do it on a test in school.

To save time on the SAT you should practice solving equations informally as much as possible. And you should also practice solving equations formally – this will increase your mathematical skill level.

Let's try another:

$$7x = 42$$

Informally, 7 times 6 is 42, so we see that $x = 6$.

Formally, we can divide each side of the equation by 7:

$$\frac{7x}{7} = \frac{42}{7}$$
$$x = 6$$

Now let's get a little harder:

$$8x + 3 = 59$$

We can still do this informally. First, let's figure out what number plus 3 is 59. Well, 56 plus 3 is 59. So, $8x$ is 56. Therefore, x must be 7.

Here is the formal solution:

$$\begin{array}{rcl} 8x + 3 & = & 59 \\ \underline{\quad -3} & & \underline{-\,3} \\ \frac{8x}{8} & = & \frac{56}{8} \\ x & = & 7 \end{array}$$

On the SAT, a combination of formal and informal algebra often works best. Let's look at an example.

LEVEL 1: HEART OF ALGEBRA – SOLVING LINEAR EQUATIONS

1. If $5 + x + x = 1 + x + x + x$, what is the value of x ?

   A) 1
   B) 2
   C) 3
   D) 4

Algebraic solution: Here is a quick algebraic solution to the problem.

$$5 + x + x = 1 + x + x + x$$
$$5 + 2x = 1 + 3x$$
$$5 = 1 + x$$
$$4 = x$$

Thus, the answer is choice **D**.

Notes: (1) For the first step, we **combined like terms**. On the left, we have $x + x = 2x$ and on the right, we have $x + x + x = 3x$.

(2) For the second step, we *formally* subtracted $2x$ from each side of the equation. Here are the details.

$$\begin{array}{rcl} 5 + 2x & = & 1 + 3x \\ \underline{\quad -2x} & & \underline{\quad -2x} \\ 5 \quad & = & 1 + \ x \end{array}$$

(3) The last step is best done *informally*. To solve $5 = 1 + x$, we are asking "5 is 1 plus what number?" Well the number is 4.

<u>A Balancing Act</u>

In this problem, we can use a little "balancing" trick to get to the answer faster. If the same quantity appears on each side of an equation, we can simply *strike it off from each side*.

*** Striking off x's:** We strike off two x's from each side of the equation to get

$$5 = 1 + x.$$

This becomes $4 = x$, choice **D**.

Notes: (1) You can physically use your pencil to strike off those x's. The marked-up problem would look as follows.

$$5 + \cancel{x} + \cancel{x} = 1 + x + \cancel{x} + \cancel{x}$$

(2) As in the previous solution, the equation $5 = 1 + x$ is best solved *informally* to save the most time.

Plug It In!

In lots of SAT math problems, you can get the answer simply by trying each of the answer choices until you find the one that works. Unless you have some intuition as to what the correct answer might be, starting in the middle with **B** or **C** is best (although there are some exceptions to this rule that we will see later). The reason for this is simple. Answers are usually given in increasing or decreasing order. If choice B or C fails, you can often eliminate one or two of the other choices as well.

Solution by starting with choice C: We start with choice C and substitute 3 in for x on each side of the equation.

$$5+3+3=1+3+3+3$$
$$11=10$$

Since this is false, we can eliminate choice C. A little thought should allow you to eliminate choices A and B as well (don't worry if you don't see this – just take another guess). Let's try choice D next.

$$5+4+4=1+4+4+4$$
$$13=13$$

Thus, the answer is choice **D**.

Notes: (1) We can begin with an algebraic solution, and then switch to plugging in later. For example, in this problem, we can write $5+2x=1+3x$, and then start substituting in the answer choices from here.

(2) The method of "plugging it in" illustrated here is usually *not* the most efficient method for solving math problems on the SAT. However, it is a *reliable* method that can be used to solve many types of multiple choice questions of varying difficulties. Keep this as a *fallback* method for when you get stuck.

UNI SAYS...

Plugging in Answer Choices is a nice method to keep around just in case you get stuck. Practice this method often, so it's easy to recall when you need it.

Time to Practice

LEVEL 1: HEART OF ALGEBRA – SOLVING LINEAR EQUATIONS

$$x+x+6x-6=5+4+2x+x+x+x$$

2. In the equation above, what is the value of x ?

A) 5
B) $\frac{15}{4}$
C) $-\frac{2}{3}$
D) -5

*** Algebraic solution:** Let's start by striking off two x's from each side of the equation to get

$$6x - 6 = 5 + 4 + 2x + x$$

We now combine like terms on the right to get $6x - 6 = 9 + 3x$. We then add 6 to each side of the equation and subtract $3x$ from each side of the equation to get $3x = 15$. Finally, we divide each side of this last equation by 3 to get $x = \frac{15}{3} = 5$, choice **A**.

Notes: (1) We can use any of the three methods of solution shown in problem 1 above, or a combination of those methods to solve this problem.

(2) Observe how the solution above uses a combination of the techniques we just learned to get to the answer as quickly as possible. We started by *striking off x's*. We then *combined like terms*. And then we finished with some *formal algebra*.

(3) It would be slightly more efficient to solve the last equation $3x = 15$ informally.

3. If $2j = \frac{x-4}{3}$ and $j = 6$, what is the value of x ?

   A) 10
   B) 20
   C) 30
   D) 40

*** Algebraic solution:** Since $j = 6$, we have $2j = 2 \cdot 6 = 12$, and so $\frac{x-4}{3} = 12$. After multiplying each side of this equation by 3, we see that $x - 4 = 3 \cdot 12 = 36$. So, $x = 36 + 4 = 40$, choice **D**.

Note: Once we have $\frac{x-4}{3} = 12$, we can solve the equation informally in two steps. Since 36 divided by 3 is 12, we have $x - 4 = 36$. Finally, $40 - 4 = 36$, and so $x = 40$.

Solution by starting with choice C: Since $j = 6$, we have that $2j = 2 \cdot 6 = 12$, and so $\frac{x-4}{3} = 12$.

Let's start with choice C and guess that $x = 30$. We then have $\frac{x-4}{3} = \frac{30-4}{3} = \frac{26}{3} \approx 8.67$. This is a bit too small and so the answer must be choice **D**.

Note: Let's just verify that choice D is correct. We replace x by 40 to get $\frac{x-4}{3} = \frac{40-4}{3} = \frac{36}{3} = 12$. This is correct, and so the answer is in fact choice D.

4. For what value of x is $\frac{5x}{2} - 7 = 23$?

*** Algebraic solution:** We add 7 to each side of the equation to get $\frac{5x}{2} = 23 + 7 = 30$. Multiplying each side of this equation by $\frac{2}{5}$ gives $x = 30 \cdot \frac{2}{5} = \frac{30}{5} \cdot 2 = 6 \cdot 2 = \mathbf{12}$.

Notes: (1) We begin by keeping all terms with an x on the left-hand side of the equation, and moving any terms without an x to the right-hand side. To do this we add 7 to each side of the equation.

(2) The **reciprocal** of $\frac{5}{2}$ is $\frac{2}{5}$. The product of reciprocals is 1. In this case, $\frac{2}{5} \cdot \frac{5}{2} = 1$.

It follows that we can solve the equation $\frac{5x}{2} = 30$ for x formally by multiplying each side of the equation by $\frac{2}{5}$.

$$\frac{3}{7}x = \frac{4}{3}$$

5. What value of x is the solution of the equation above?

A) $\frac{4}{7}$
B) $\frac{9}{7}$
C) $\frac{28}{9}$
D) $\frac{28}{3}$

*** Algebraic solution:** We multiply each side of the equation by $\frac{7}{3}$ to get $x = \frac{4}{3} \cdot \frac{7}{3} = \frac{28}{9}$, choice **C**.

Note: See problem 4 above for more information about using reciprocals to solve equations.

Solution by starting with choice C: Let's start with choice C and substitute $\frac{28}{9}$ in for x on the left-hand side of the equation. We then have $\frac{3}{7} \cdot \frac{28}{9} = \frac{3}{9} \cdot \frac{28}{7} = \frac{1}{3} \cdot \frac{4}{1} = \frac{4}{3}$. Since this is equal to the right-hand side, the answer is choice **C**.

Level 2: Heart of Algebra – Solving Linear Equations

$$L = 11 + 1.6M$$

6. One end of an elastic band is taped to the bottom of a ceiling fan. When an object of mass M kilograms is attached to the other end of the elastic band, the band stretches to a length of L centimeters as shown in the equation above. What is M when $L = 13$?

*** Algebraic solution:** We let $L = 13$ and solve for M.

$$13 = 11 + 1.6M$$
$$2 = 1.6M$$
$$M = \frac{2}{1.6} = \frac{20}{16} = \mathbf{5/4} \text{ or } \mathbf{1.25}.$$

Note: There are lots of words in this problem that are completely unnecessary. The question could have simply been asked as follows:

If $L = 11 + 1.6M$, what is M when $L = 13$?

The SAT often likes to artificially create a "real world" context which is not needed when solving the problem.

7. If $4x - 5 = 53$, what is the value of $12x - 2$?

*** Algebraic solution:** We add 5 to each side of the given equation to get $4x = 53 + 5 = 58$. We now multiply each side of this last equation by 3 to get $12x = 174$. Finally, we subtract 2 from each side of this last equation to get $12x - 2 = 174 - 2 = \mathbf{172}$.

Notes: (1) We did *not* need to find x to solve this problem. Since the expression we are trying to find has $12x$ as one of its terms, it is more efficient to multiply $4x$ by 3, than it is to solve the original equation for x and then multiply by 12.

(2) If you didn't notice that you could change $4x$ into $12x$ by multiplying by 3, then it's not too big of a deal. The problem could still be solved easily by solving for x first. Once we have $4x = 58$, we can divide each side of the equation by 4 to get $x = \frac{58}{4}$. We then have

$$12x - 2 = 12 \cdot \frac{58}{4} - 2 = \frac{12}{4} \cdot 58 - 2 = 3 \cdot 58 - 2 = 174 - 2 = 172$$

LEVEL 3: HEART OF ALGEBRA – SOLVING LINEAR EQUATIONS

8. If $15x = 73$, what is the value of $3(x + \frac{4}{5})$?

A) 17
B) 15
C) $\frac{73}{15}$
D) $\frac{77}{15}$

Algebraic solution: We divide each side of the given equation by 15 to get $x = \frac{73}{15}$. It follows that $3(x + \frac{4}{5}) = 3\left(\frac{73}{15} + \frac{4}{5}\right) = 17$, choice **A**.

Notes: (1) If a calculator is allowed for this problem we can simply type the following in our calculator to get the answer:

3(73/ 15 + 4 / 5) ENTER

(2) If a calculator is not allowed, then we would begin by rewriting $\frac{4}{5}$ as $\frac{4}{5} \cdot \frac{3}{3} = \frac{12}{15}$. We then have $\frac{73}{15} + \frac{4}{5} = \frac{73}{15} + \frac{12}{15} = \frac{(73+12)}{15} = \frac{85}{15}$. Then $3\left(\frac{73}{15} + \frac{4}{5}\right) = 3\left(\frac{85}{15}\right) = \left(\frac{3}{15}\right)(85) = \left(\frac{1}{5}\right)(85) = 17$.

*** Quicker algebraic solution:** We divide each side of the given equation by 5 to get $3x = \frac{73}{5}$. We then have

$$3\left(x + \frac{4}{5}\right) = 3x + 3 \cdot \frac{4}{5} = 3x + \frac{12}{5} = \frac{73}{5} + \frac{12}{5} = \frac{73 + 12}{5} = \frac{85}{5} = 17.$$

This is choice **A**.

Notes: (1) The **distributive property** says that for all real numbers a, b, and c,

$$a(b + c) = ab + ac$$

More specifically, this property says that the operation of multiplication distributes over addition.

(2) We used the distributive property in the first step of this solution. In this problem, we have $a = 3$, $b = x$, and $c = \frac{4}{5}$. Thus, we have $3\left(x + \frac{4}{5}\right) = 3x + 3 \cdot \frac{4}{5}$.

(3) See Lesson 2 for more on the distributive property.

9. On Sunday, Janice studied 3 more hours than Chris. If they studied for a combined total of 13 hours, how many hours did Chris study for on Sunday?

A) 5
B) 6
C) 7
D) 8

*** Algebraic solution:** Let x be the number of hours Chris studied. Then Janice studied $x + 3$ hours, and we have $x + (x + 3) = 13$. Therefore, $2x + 3 = 13$, and so $2x = 13 - 3 = 10$. So, the number of hours that Chris studied is $x = \frac{10}{2} = 5$, choice **A.**

Note: This problem can also be solved by plugging in the answer choices (start with choice B or C). I leave the details of this solution to the reader.

LEVEL 4: HEART OF ALGEBRA – SOLVING LINEAR EQUATIONS

10. A gymnast's final score is determined by the sum of the difficulty score and execution score, less any deductions for neutral errors. Jackie had a difficulty score of p points and an execution score of q points. Assuming that Jackie lost $\frac{1}{8}$ of a point for each of her 20 neutral errors and had a final score of 6.5, what is the value of $p + q$?

*** Algebraic solution:** We have $p + q - \frac{1}{8} \cdot 20 = 6.5$. So, $p + q = 6.5 + \frac{1}{8} \cdot 20 = 6.5 + 2.5 = \mathbf{9}$.

Notes: (1) Since Jackie's difficulty score was p and her execution score was q, it follows that the *sum* of her difficulty score and execution score was $p + q$.

(2) Jackie had 20 neutral errors and she lost $\frac{1}{8}$ of a point for each one. It follows that she lost a *total* of $d = \frac{1}{8} \cdot 20 = \frac{20}{8} = 2.5$ points due to neutral errors.

(3) Jackie's total score, T, is determined by $T = p + q - d$, where p is Jackie's difficulty score, q is Jackie's execution score, and d is Jackie's total deductions due to neutral errors.

We were given that $T = 6.5$ in the question, and we found that $d = 2.5$ in note (2). It follows that $6.5 = p + q - 2.5$. Adding 2.5 to each side of this last equation gives us $p + q = 6.5 + 2.5 = 9$.

About the Author

Dr. Steve Warner, a New York native, earned his Ph.D. at Rutgers University in Pure Mathematics in May 2001. While a graduate student, Dr. Warner won the TA Teaching Excellence Award.

After Rutgers, Dr. Warner joined the Penn State Mathematics Department as an Assistant Professor. In September 2002, Dr. Warner returned to New York to accept an Assistant Professor position at Hofstra University. By September 2007, Dr. Warner had received tenure and was promoted to Associate Professor. He has taught undergraduate and graduate courses in Precalculus, Calculus, Linear Algebra, Differential Equations, Mathematical Logic, Set Theory and Abstract Algebra.

Over that time, Dr. Warner participated in a five-year NSF grant, “The MSTP Project,” to study and improve mathematics and science curriculum in poorly performing junior high schools. He also published several articles in scholarly journals, specifically on Mathematical Logic.

Dr. Warner has more than 15 years of experience in general math tutoring and tutoring for standardized tests such as the SAT, ACT and AP Calculus exams. He has tutored students both individually and in group settings.

In February 2010 Dr. Warner released his first SAT prep book “The 32 Most Effective SAT Math Strategies,” and in 2012 founded Get 800 Test Prep. Since then Dr. Warner has written books for the SAT, ACT, SAT Math Subject Tests, AP Calculus exams, and GRE.

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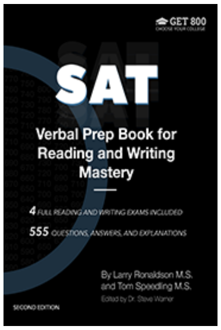

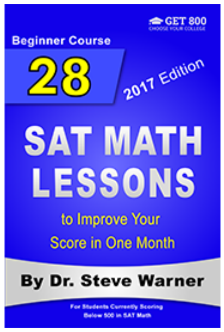

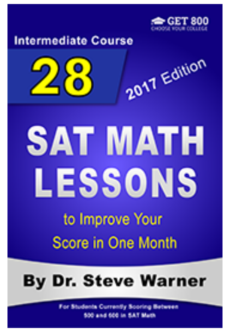

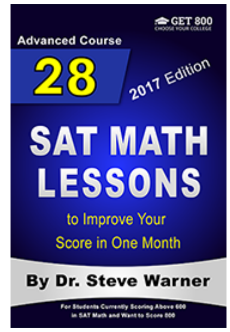

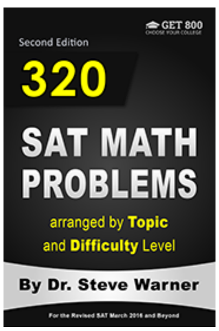

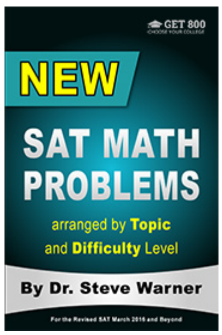

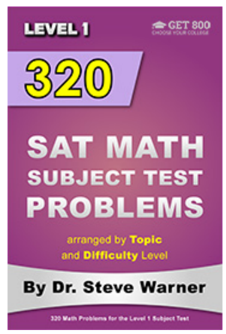

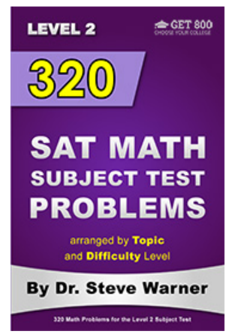

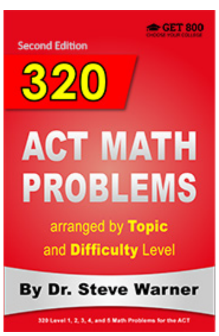

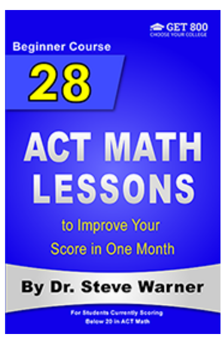

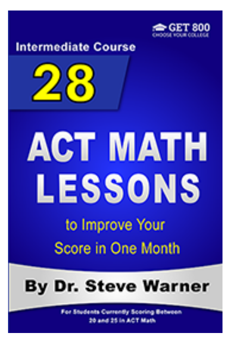

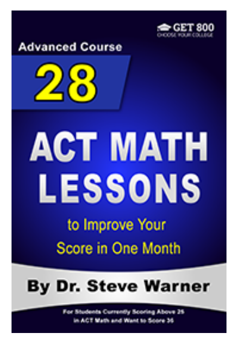

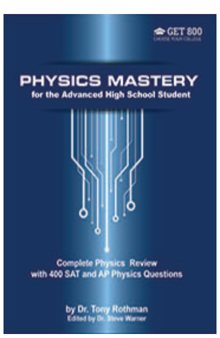

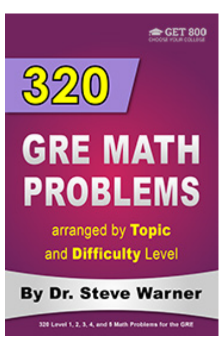

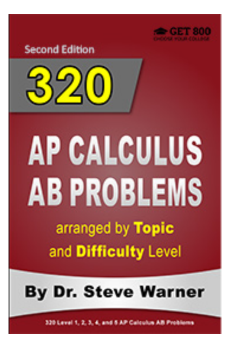

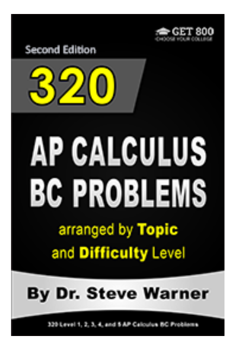

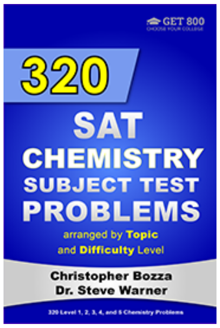

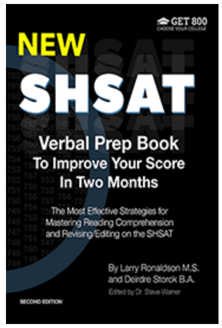

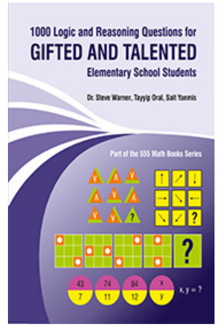

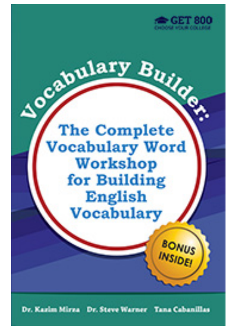

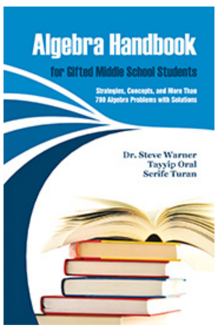

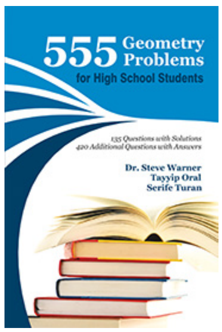

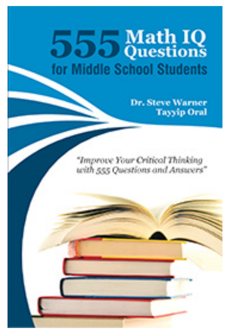

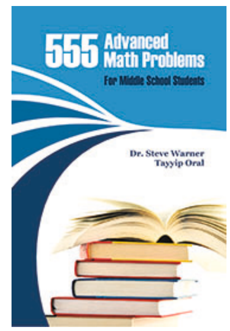

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The Scholarly Unicorn's SAT Math Advanced Guide with 1000 Problems and 48 Lessons was written by a Ph.D. in mathematics to help students achieve a perfect SAT math score.

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Each of the 1000 problems in this book comes with at least one complete explanation (and often more) followed by helpful remarks to ensure that you develop a deep understanding of all the material presented.

Here is what customers say about Dr. Warner's previous work:

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