800 INITIATION PROBLEM SET FOR PART 12

LEVEL 1: NUMBER THEORY

- 1. Which of the following numbers is less than 0.736?
 - (A) 0.7306
 (B) 0.7361
 (C) 0.7366
 (D) 0.74
 (E) 0.741

Beginner Method: We can compare two decimals by looking at the first position where they disagree. For example, 0.7306 is less than 0.7361 because 0 is less than 6. If a digit is missing, there is a hidden 0 there. Thus 0.74 is less than 0.741 because 0.74 is the same as 0.740 and 0 is less than 1. Thus the answer here is choice (A).

* **Quick Solution:** Answers are almost always given in increasing or decreasing order on the SAT (in this problem they are given in increasing order). Therefore choice (A) is the only reasonable answer.

Remark: The words "less than" would seem to indicate we should start by looking at the smallest answer choice. In this case that is choice (A).

LEVEL 3: NUMBER THEORY

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- 2. In the pattern above, the first letter is B and the letters B, U, G, L and E repeat continually in that order. What is the 124th letter in the pattern?
 - (A) B
 - (B) U
 - (C) G
 - (D) L
 - (E) E

Beginner Method: We first find an integer as close as possible to 124 that is divisible by 5 (there are five letters in the pattern). We can check this in our calculator. We find that 125 is divisible by 5. Thus, the 125th letter is E. So the 124th letter is L, choice (D).

* Advanced Method: The remainder upon dividing 124 by 5 is 4. Therefore the 124th letter in the sequence is the same as the 4th letter in the sequence, which is L.

Caution: If the remainder is 0, you get the 5th letter in the sequence (there is no 0th letter). For example, the 125th letter in the sequence is E (as mentioned in the solution above).

Calculator Algorithm for computing a remainder: Although performing division in your calculator never produces a remainder, there is a simple algorithm you can perform which mimics long division. Let's find the remainder when 124 is divided by 5 using this algorithm.

Step 1: Perform the division in your calculator: 124/5 = 24.8

Step 2: Multiply the integer part of this answer by the divisor: 24.5 = 120

Step 3: Subtract this result from the dividend to get the remainder:

124 – 120 = **4**.

For more information take a look at this blog post:

http://satprepget800.com/2013/03/18/sat-math-remainder-problems/

LEVEL 1: ALGEBRA AND FUNCTIONS

- 3. If 3y 18 = 15, then y 6 =
 - (A) 30
 (B) 20
 (C) 15
 (D) 10
 (E) 5

Beginner Method: Let's start with a "random" guess for y, say y = 10. So let's plug 10 in for y in the first equation.

$$3y - 18 = 15$$

 $3 \cdot 10 - 18 = 15$
 $30 - 18 = 15$
 $12 = 15$

It looks as though 10 is a little too small. y = 11 should do the trick.

$$3y - 18 = 15$$

 $3 \cdot 11 - 18 = 15$
 $33 - 18 = 15$
 $15 = 15$

So y is, in fact, 11. Thus, y - 6 = 11 - 6 = 5, and the answer is choice (E).

For more information on this technique take a look at the following blog post:

http://satprepget800.com/2013/01/06/basic-sat-math-strategy-take-a-guess/

Algebraic solution: We solve for *y* algebraically.

So y - 6 = 11 - 6 = 5, and the answer is choice (E).

* Quicker algebraic solution: We solve for y - 6 algebraically.

Thus, the answer is choice (E).

LEVEL 3: ALGEBRA AND FUNCTIONS

4. If x is $\frac{3}{5}$ of y and y is $\frac{5}{7}$ of z, what is the value of $\frac{z}{x}$?

(A)
$$\frac{1}{4}$$

(B) $\frac{3}{7}$
(C) $\frac{5}{4}$
(D) $\frac{10}{7}$
(E) $\frac{7}{3}$

* **Beginner Method:** Let's choose a value for z, say z = 35 (this seems like a good choice since it is the product of the two denominators). Then

 $y = \frac{5}{7} \cdot 35 = 25$ (the word "of" indicates multiplication),

and $x = \frac{3}{5} \cdot 25 = 15$. Therefore, $\frac{z}{x} = \frac{35}{15} = \frac{7}{3}$, choice (E).

Recall: To reduce the fraction $\frac{35}{15}$, type $\frac{35}{15}$ in your calculator, then press MATH ENTER ENTER.

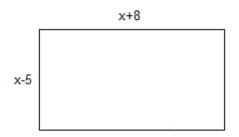
For more information on this technique take a look at the following blog post:

http://satprepget800.com/2013/01/27/pick-a-number-solve-the-problem/

Algebraic solution: $x = \frac{3y}{5}$ and $y = \frac{5z}{7}$. Solving the last equation for z gives us $z = \frac{7y}{5}$. Then $\frac{z}{x} = (\frac{7y}{5})/(\frac{3y}{5}) = \frac{7y}{5} \cdot \frac{5}{3y} = \frac{7}{3}$.

Thus, the answer is choice (E).

LEVEL 1: GEOMETRY



- 5. If the perimeter of the rectangle above is 78, what is the value of x?
 - (A) 20
 (B) 19
 (C) 18
 (D) 17
 (E) 16

* **Beginner Method:** Recall that we get the **perimeter** of a rectangle by adding up all four sides. Let's start with choice (C) as our first guess, so that x = 18. It follows that x - 5 = 18 - 5 = 13 and x + 8 = 18 + 8 = 26. So the perimeter of the rectangle is 13 + 13 + 26 + 26 = 78. Therefore the answer is choice (C). For more information on this technique take a look at the following blog post:

http://satprepget800.com/2013/10/28/sat-strategy-video-starting-with-choice-c/

Algebraic solution: We solve the following equation.

$$P = 2\ell + 2w$$

$$78 = 2(x + 8) + 2(x - 5)$$

$$78 = 2x + 16 + 2x - 10$$

$$78 = 4x + 6$$

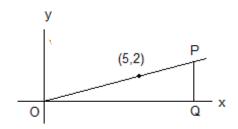
$$72 = 4x$$

$$\frac{72}{4} = x$$

$$18 = x$$

This is answer choice (C).

LEVEL 3: GEOMETRY



6. Line k (not shown) passes through O and intersects \overline{PQ} between P and Q. What is one possible value of the slope of line k?

* **Beginner method:** Let's choose a specific line k. The easiest choice is the line passing through (0, 0) and (5, 1). Now plug these two points into the slope formula to get $\frac{1-0}{5-0} = 1/5$.

Remarks: (1) Here we have used the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

(2) If the line *j* passes through the origin (the point (0, 0)) and the point (*a*, *b*) with $a \neq 0$, then the slope of line *j* is simply $\frac{b}{a}$.

Advanced method: The slope of line \overline{OP} is $\frac{2}{5} = .4$ (see Remark (2) above) and the slope of line \overline{OQ} is 0. Therefore we can choose any number strictly between 0 and .4 that fits in the answer grid.

LEVEL 1: PROBABILITY AND STATISTICS

7. In a jar there are exactly 63 marbles, each of which is yellow, purple, or blue. The probability of randomly selecting a yellow marble from the jar is $\frac{4}{7}$, and the probability of randomly selecting a purple marble from the jar is $\frac{1}{7}$. How many marbles in the jar are blue?

Straightforward solution: The number of yellow marbles in the jar is $(\frac{4}{7})(63) = 36$. The number of purple marbles in the jar is $(\frac{1}{7})(63) = 9$. Therefore the number of blue marbles in the jar is (63 - 36 - 9) = 18.

* Quick solution: The probability of selecting a blue marble from the jar is $1 - \frac{4}{7} - \frac{1}{7} = \frac{2}{7}$. Therefore the number of blue marbles in the jar is $(\frac{2}{7})(63) = 18$.

Remark: The probability of a **certain event** is 1. In particular, if you add up the probabilities of each possible outcome you get 1.

LEVEL 3: PROBABILITY AND STATISTICS

8. Set X contains only the integers 0 through 140 inclusive. If a number is selected at random from X, what is the probability that the number selected will be less than 93?

* There are a total of 141 integers. There are 93 integers from set X that are less than 93. Therefore the probability is $\frac{93}{141}$ which is approximately .6595744681. So we can grid in .659 or .660.

Remark: The number of integers from *a* to *b*, inclusive, is b - a + 1. So in this example, the number of integers from 0 to 160 is 160 - 0 + 1 = 161, and the number of integers from 0 to 82 is 82 - 0 + 1 = 83. If you ever forget this little formula, test it out on a small list. For example, let's count the number of integers from 3 to 7, inclusive. They are 3, 4, 5, 6, 7, and we see there are 5 of them. Now 7 - 3 = 4 which is not the correct amount, but 7 - 3 + 1 = 5 which is the correct amount.

For more information on the formula used here take a look at the following blog post:

http://satprepget800.com/2013/05/20/sat-math-count-integers/

LEVEL 2: NUMBER THEORY

 $\frac{3}{n}, \frac{21}{n}, \frac{33}{n}$

- 9. If each of the fractions above is in its simplest reduced form, then which of the following could be the value of n?
 - (A) 15
 - (B) 32
 - (C) 42
 - (D) 66
 - (E) 77

Beginner method: A fraction is in simplest reduced form if the numerator (top) and denominator (bottom) have no common factors greater than 1. For example $\frac{3}{42}$ is not reduced since 3 and 42 are both divisible by 3. This eliminates choice (C). Since 3 and 66 are both divisible by 3 we can eliminate choice (D) as well. Since 32 has no factors in common with 3, 21 or 33 we see that choice (B) is the answer.

For more information on this technique take a look at the following blog post:

http://satprepget800.com/2013/10/28/sat-strategy-video-starting-with-choice-c/

* Advanced method: 3, 21 and 33 have prime factors of 3, 7 and 11. So we simply pick the answer choice whose prime factorization does not consist of any of these integers. Since the only prime factor of 32 is 2, choice (B) is the answer.

For more about primes take a look here:

http://satprepget800.com/2013/06/03/number-definitions-sat-math/

LEVEL 3: NUMBER THEORY

10. If an integer *n* is divisible by 3, 7, 21, and 49, what is the next larger integer divisible by these numbers?

(A) n + 21
(B) n + 49
(C) n + 73
(D) n + 147
(E) n + 294

Beginner method: Let's choose a value for *n* satisfying the given condition. If we multiply the given numbers together, then we get n = 24,696. Starting with choice (A) plug in 24,696 for *n*, and divide the result by each of the given four numbers.

- (A) 24,696 + 21 = 24,717 (not divisible by 49: 24,717/49 ~ 504.4)
- (B) 24,696 + 49 = 24,745 (not divisible by 3: 24,745/3 ~ 8248.33)
- (C) 24,696 + 73 = 24,769 (not divisible by 3: 24,769/3 ~ 8256.33)
- (D) 24,696 + 147 = 24,843 (divisible by all 3)

Since (D) works we can stop here and choose answer choice (D).

Notes:

- (a) We only need to check divisibility by 3 and $7^2 = 49$ since these are the highest powers of primes that are factors of the given numbers.
- (b) 21 and 49 would work as well (since together they contain the factors 3 and $7^2 = 49$).
- (c) A better choice for n is the **least common multiple** of the four given numbers which is $3 \cdot 7^2 = 147$. In this case we get the following:
 - (A) 147 + 21 = 168 (not divisible by 49: 168/49 ~ 3.4)
 - (B) 147 + 49 = 196 (not divisible by 3: 196/3 ~ 65.33)
 - (C) 147+73 = 220 (not divisible by all 3: 220/3 ~ 73.33)
 - (D) 147+ 147 = 294 (divisible by all 3)

For more information on this technique take a look at the following blog post:

http://satprepget800.com/2013/01/27/pick-a-number-solve-the-problem/

Advanced method: As stated in note (c) above, the least common multiple of the given numbers is 147. We can therefore add any multiple of 147 to *n* and maintain divisibility by each of the 4 given numbers. So choice (D) is the correct answer.

Note: Choice (E) also always gives an integer divisible by the given 4 numbers. It is not correct because it is not the **next** larger integer.

Remarks: (1) Note that if *n* is divisible by 147 it can be written as 147k for some integer *k*. Thus n + 147 = 147k + 147 = 147(k + 1). So n + 147 is divisible by 147, and thus by any factor of 147 including 3, 7, 21 and 49.

(2) Now that we know the above theory we see that we can get the next larger number divisible by the given numbers by adding the lcm of the given numbers.

* Quick Solution: lcm = $3 \cdot 7^2$ = 147. So the answer is n + 147, choice (D).

For more about the lcm check here:

http://satprepget800.com/2013/06/09/sat-math-lcm-gcm/

LEVEL 2: ALGEBRA AND FUNCTIONS

11. If x + 7y = 15 and x + 3y = 7, what is the value of x + 5y?

* We add the two equations to get

Now observe that 2x + 10y = 2(x + 5y). So $x + 5y = \frac{22}{2} = 11$.

For more information on this technique, you may want to watch this video:

http://www.youtube.com/watch?v=zXazG5KR1h0

LEVEL 3: ALGEBRA AND FUNCTIONS

6 <i>z</i>	2 <i>z</i>
8	8
2 <i>t</i>	W
3	3
<u>+9</u>	<u>+9</u>
52	34

12. In the correctly worked addition problems above, what is the value of 4z + 2t - w?

Solution using a simple operation: Let's rewrite the equations horizontally since that is how most of us are used to seeing equations.

$$5z + 4 + 4t + 8 + 9 = 52$$

 $2z + 4 + w + 8 + 9 = 34$

We now use a simple operation. The operation to use here is subtraction. Let's go ahead and subtract term by term.

$$5z + 4 + 4t + 8 + 9 = 52$$

$$2z + 4 + w + 8 + 9 = 34$$

$$3z + (4t - w) = 18$$

Remark: Whenever we are trying to find an expression that involves addition, subtraction, or both, **adding or subtracting** the given equations usually does the trick.

Once again, watch this video for more information:

http://www.youtube.com/watch?v=zXazG5KR1h0

* Visualizing the answer: You can save a substantial amount of time by performing the subtraction in your head (left equation minus right equation). Note that above the lines the subtraction yields 3z + 4t - w. This is exactly what we're looking for. Thus, we need only subtract below the lines to get the answer: 52 - 34 = 18.

Solution by picking numbers: If we choose any value for *z*, then *t* and *w* will be determined. So, let's set *z* equal to 0. Then

$$4 + 4t + 8 + 9 = 52$$

$$4t + 21 = 52$$

$$4t = 31$$

$$t = \frac{31}{4} = 7.75$$

$$4 + w + 8 + 9 = 34$$

$$w + 21 = 34$$

$$w = 13$$

and

So
$$3z + 4t - w = 0 + 4(7.75) - 13 = 18$$
.

Remarks: (1) Any choice for z will give us the same answer. We could have chosen a value for t or w as well. But once we choose a value for one of the variables the other two are determined.

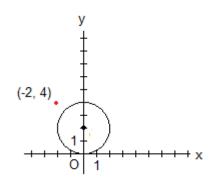
(2) It was actually unnecessary to solve for t above. We could have stopped at 4t = 31. We then have 3z + 4t - w = 0 + 31 - 13 = 18.

For more information on this technique take a look at the following blog post:

http://satprepget800.com/2013/01/27/pick-a-number-solve-the-problem/

LEVEL 2: GEOMETRY

- 13. In the xy-plane, the point (0, 2) is the center of a circle that has radius 2. Which of the following is NOT a point on the circle?
 - (A) (0,4)
 (B) (-2,4)
 (C) (2,2)
 (D) (-2,2)
 (E) (0,0)
- * Let's begin by drawing a picture.



From the picture it should be clear that (-2, 4) is not on the circle. This is choice (B).

LEVEL 3: GEOMETRY

14. How many spherical snowballs with a radius of 4 centimeters can be made with the amount of snow in a spherical snowball of radius 8 centimeters? (the volume V of a

sphere with radius *r* is given by $\frac{4}{3}\pi r^3$.)

* We divide the volumes.

$$\frac{\frac{4}{3}\pi \cdot 8^3}{\frac{4}{3}\pi \cdot 4^3} = \frac{8^3}{4^3} = \frac{512}{64} = \mathbf{8}.$$

LEVEL 2: PROBABILITY AND STATISTICS

3, 5, 6, 15, 27, 35, 45, 75

15. A number is to be selected at random from the list above. What is the probability that the number selected will be a multiple of both 9 and 5?

* The total number of outcomes is 8. The number of "successes" is 1. Therefore the probability is **1/8** or **.125**.

Notes:

(1) In this problem a "success" is a number that is a multiple of both 9 and 15.

(2) You can check if each number is a "success" by dividing the number by 9 and by 15. If you get an integer in both cases, then the number is a "success." Otherwise it is not. For example, 27/9 = 3 and 27/15 = 1.8. Since 1.8 is not an integer, the number 27 is <u>not</u> a "success."

(3) You can check if each number is a "success" more quickly by dividing the number by 45. If you get an integer, then the number is a "success." Otherwise it is not. For example, 27/45 = .6 which is not an integer. Therefore the number 27 is <u>not</u> a "success."

(4) The only "success" here is 45.

LEVEL 3: COUNTING

16. A chemist is testing 9 different liquids. For each test, the chemist chooses 4 of the liquids and mixes them together. What is the least number of tests that must be done so that every possible combination of liquids is tested?

* Solution using combinations: We are counting the number of ways to choose 4 of the 9 liquids. This is ${}_{9}C_{4} = 126$.

Remarks:

(1) This is a combination because it does not matter what order we take the three liquids. We are simply grabbing four of them and mixing them together.

(2) We can compute ${}_{9}C_{4}$ very quickly on our calculator as follows: first type 9. Then under the Math menu scroll over to PRB and select **nCr**. Finally type 3 and press ENTER.

(3) The formula for ${}_{n}C_{r}$ is $\frac{n!}{r!(n-r)!}$. So ${}_{9}C_{4} = \frac{9!}{5!4!} = \frac{(9)(8)(7)(6)}{(4)(3)(2)(1)} = 126$. (Note that this is included for completeness. You do not need to know this formula.)

For more information on combinations take a look at the following blog post:

http://satprepget800.com/2013/08/19/sat-counting-3/

LEVEL 2: NUMBER THEORY

- 17. In September, Maria was able to type 30 words per minute. In October she was able to type 42 words per minute. By what percent did Jennifer's speed increase from September to October?
 - (A) 12%
 - (B) 18%
 - (C) 30%
 - (D) 40%
 - (E) 42%

* This is a percent increase problem. So we will use the formula for percent change.

$$Percent \ Change = \frac{Change}{Original} \times 100$$

The **original** value is 30. The new value is 42, so that the **change** is 12. Using the percent change formula, we get that the percent increase is $\frac{12}{30} \cdot 100 = .4 = 40\%$, choice (D).

Warning: Do not accidently use the new value for "change" in the formula. The **change** is the positive difference between the original and new values.

LEVEL 3: NUMBER THEORY

- 18. The ratio of the number of elephants to the number of zebras in a zoo is 3 to 5. What percent of the animals in the zoo are zebras?
 - (A) 12.5%
 - (B) 37.5%
 - (C) 60%
 - (D) 62.5%
 - (E) 70%

We can represent the number of elephants in the zoo by 3x and the number of zebras in the zoo by 5x for some number x. Then the total number of animals in the zoo is 8x which we set equal to 100. Now 8x = 100 implies that $x = \frac{100}{8} = 12.5$. Since we want the percent of the animals in the park that are zebras, we need to find 5x = 5(12.5) = 62.5%, choice (D).

Important note: After you find x make sure you look at what the question is asking for. A common error is to give an answer of 12.5%. But the number of zebras is **not** equal to x. It is equal to 5x.

* Alternate solution: We set up a ratio of the amount of zebras in the zoo to the total number of animals in the zoo.

zebras 5 x
animals 8 100
$$\frac{5}{8} = \frac{x}{100}$$
$$8x = 500$$
$$x = \frac{500}{8} = 62.5$$
, choice (D).

LEVEL 3: FUNCTIONS

19. The function p is defined by $p(x) = 3x^2 - cx - 5$, where c is a constant. In the xyplane, the graph of y = p(x) crosses the x-axis where x = 2. What is the value of c?

* A graph crosses the *x*-axis at a point where y = 0. Therefore the point (2, 0) is on the graph of y = p(x). Equivalently, p(2) = 0. So

 $0 = 3(2)^{2} - 2c - 5$ 0 = 3(4) - 2c - 5 0 = 12 - 2c - 5 0 = 7 - 2c 2c = 7c = 7/2 or 3.5

LEVEL 3: GEOMETRY

- 20. Which of the following is an equation of the line in the *xy*-plane that passes through the point (0, -3) and is perpendicular to the line y = -4x + 7?
 - (A) y = -4x 6(B) y = -4x - 3(C) y = -4x + 3(D) $y = \frac{1}{4}x - 3$ (E) $y = \frac{1}{4}x + 6$

* **Solution by plugging in the point:** Since the point (0,-3) lies on the line, if we substitute 0 in for *x*, we should get -3 for *y*. Let's substitute 0 in for *x* in each answer choice.

(A) -6
(B) -3
(C) 3
(D) 3
(E) 6

We can eliminate choices (A), (C), (D) and (E) because they did not come out to -3. The answer is therefore choice (B).

Important note: (B) is **not** the correct answer simply because *y* came out to -3. It is correct because all 4 of the other choices did **not** give -3 for *y*.

Geometric solution: Recall the slope-intercept form for the equation of a line.

y = mx + b

(0,-3) is the y-intercept of the point. Thus, b = -3. The slope of the given line is -4. Since the new line is parallel to this line, its slope is also -4, and the equation of the new line is y = -4x - 3, choice (B).

Recall: Parallel lines have the same slope, and perpendicular lines have slopes that are negative reciprocals of each other.

LEVEL 3: STATISTICS

21. The average of x, y, z, and w is 12 and the average of z and w is 7. What is the average of x and y?

* Solution using the formula Sum = Average \cdot Number: The Sum of x, y, z, and w is $12 \cdot 4 = 48$. The Sum of z and w is $7 \cdot 2 = 14$. Thus, the Sum of x and y is 48 - 12 = 34. Finally, the Average of x and y is $\frac{34}{2} = 17$.

Notes: (1) We used the formula "Sum = Average · Number" twice here. (2) More formally we have the following.

$$x + y + z + w = 48$$
$$\frac{z + w = 14}{x + y} = 34$$

Thus, $\frac{x+y}{2} = \frac{36}{2} = 17$.

For more information on this technique take a look at the following blog post:

http://satprepget800.com/2014/01/13/sat-math-strategy-change-averages-to-sums/

Solution by picking numbers: Let's let z = w = 7 and x = y = 17. Note that the average of x, y, z, and w is 12 and the average of z and w is 7. Now just observe that the average of x and y is 17.

Remarks: (1) If all numbers in a list are all equal, then the average of these numbers is that number as well.

(2) When choosing numbers to form a certain average, just "balance" these numbers around the average. In this example we chose z and w to be 7. Since 7 is 5 less than the average, we chose x and y to be 5 greater than the average.

For more information on this technique take a look at the following blog post:

http://satprepget800.com/2013/01/27/pick-a-number-solve-the-problem/

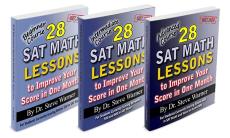
Complete algebraic solution: We are given that $\frac{x+y+z+w}{4} = 12$ and $\frac{z+w}{2} = 7$. We multiply each side of the first equation by 4 and each side of the second equation by 2 to eliminate the denominators. Then we subtract the second equation from the first.

$$x + y + z + w = 48$$
$$\frac{z + w = 14}{x + y} = 34$$

Finally, the average of x and y is $\frac{x+y}{2} = \frac{34}{2} = 17$.

Important note: You should avoid this method on the actual SAT. It is too time consuming.

Well that's it for the problem sets from the Get 800 Initiation Course. Remember to keep redoing each of these problems until you can get each one correct on your own. If you would like to continue learning strategies and practicing SAT math problems, please check out <u>"28 SAT Math Lessons To Improve Your Score In One Month."</u> As a student currently scoring below 500 in SAT math on College Board practice tests, you are probably most interested in the <u>Beginner Course</u>.



Best of luck, Dr. Steve



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